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Robust multi-criteria sorting with the outranking preference model and characteristic profiles

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ABSTRACT

We present a new multiple criteria sorting approach that uses characteristic profiles for defining the classes and outranking relation as the preference model, similarly to the Electre Tri-C method. We reformulate the conditions for the worst and best class assignments of Electre Tri-C to increase comprehensibility of the method and interpretability of the results it delivers. Then, we present a disaggregation procedure for inferring the set of outranking models compatible with the given preference information, and use the set in deriving, for each decision alternative, the necessary and possible assignments. Furthermore, we introduce simplified assignment procedures and prove that they maintain a no class jumps-property in the possible assignments. Application of the proposed approach is demonstrated by classifying 40 land zones in 4 classes representing different risk levels.

1. Introduction

In the multiple criteria sorting problem (also called ordinal classification), decision alternatives are assigned to one or more homogeneous classes based on their evaluations on multiple attributes. The classes considered here are ordered and pre-defined, which means that they, unlike clusters [1], do not result from the analysis. For example, the submitted papers need to be assigned to categories reject, weak reject, weak accept, or accept, and employees may be acknowledged for their performance with a low, medium or high bonus. Indeed, sorting is an important decision problem in fields such as finance [2,3] and environmental risk assessment and management [4–6].

In this paper we consider multi-criteria sorting problems applying the non-compensatory outranking preference model (for some recent advances and applications of outranking-based approaches see [7–11]). The most well-known among such methods is Electre Tri-B [12] that employs boundary profiles for modeling the frontiers between two consecutive classes. However, in some decision situations the frontiers between the classes have no objective existence because the separation between the consecutive classes can be conceived in several ways. In Electre Tri-C [13] the alternatives are not compared against the class boundaries, but rather with characteristic profiles that are formed from the representative attribute values for each class. For each decision alternative, Electre Tri-C results in an assignment in form of an interval of classes. The aim of the current paper is to make Electre Tri-C more usable in real-life analyses by introducing the following four advances.

First of all, we reformulate the two Electre Tri-C assignment procedures whose outcomes delimit a resulting interval of classes for each alternative. Depending on the results of a comparison of an alternative with the characteristic profiles, the order of classes indicated by the descending and ascending assignment rules may vary. That is, with some outranking models the ascending rule may indicate an assignment to a better class than the descending rule, whereas with other models the order can be reversed. While respecting the assumptions and providing the same results as Electre Tri-C, the reformulated assignment procedures indicate the lower and upper classes unambiguously. Discovering the precise conditions for the extreme class assignments is important for two reasons. First, it increases the transparency of the method. Second, the reformulated procedures support the explanation of the results in natural language, thus increasing the comprehensibility of the sorting recommendation.

The second aim of the paper is to introduce a disaggregation procedure for inferring the parameters for the preference model used in Electre Tri-C. As in other sorting methods that apply the outranking preference model, it is not realistic to assume that the Decision Maker (DM) can provide exact values for all the parameters. Some of them can be defined by the DM fairly easily

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(comparison thresholds, class profiles) [12], but others (criteria weights and the majority threshold) are harder to elicit [14–16]. Although Electre Tri-C has been found appropriate for the context of real-world problems in project management [17], environmental modeling and assessment [18,19], tourism [20], and medical diagnosis [21], most of these studies report problems settings with precise values for the inter-criteria model parameters (for discussion on eliciting precise values for the weights and the majority threshold in Electre Tri-C, see, respectively, [21] and [17]).

We generalize the disaggregation procedure to account for imprecise assignment examples. In this way, we provide the first disaggregation approach for outranking-based sorting methods that admits specification of interval assignments. This increases the flexibility of the preference elicitation process by allowing preference statements reflecting hesitation with respect to the desired assignment (e.g., "*a* should be assigned to class medium or good") as well as these concerning classes excluded from the set of desired assignments (e.g., "*b* should not be assigned to class bad"). The main challenge in designing a disaggregation procedure for Electre Tri-C is that its assignment rules indicate an interval of classes. Due to this imprecision the conditions for assigning an alternative a to a certain interval of classes $[C_{L^{DM}}, C_{R^{DM}}]$, $L^{DM} \leq R^{DM}$, cannot be derived directly from the respective sorting rules. Instead, one must guarantee that a is assigned neither to a class worse than $C_{L^{DM}}$ nor better than $C_{R^{DM}}$.

Our procedures infer a set of Electre Tri-C models from assignment examples. To avoid an arbitrary definition or selection of a single model for the final classification, the third aim of the paper involves the adaptation of the Robust Ordinal Regression (ROR) methodology [16,22–25] to outranking-based multiple criteria sorting. By exploiting the whole set of compatible Electre Tri-C models, we compute the necessary and possible assignments for each alternative, that is, assignments that hold for all or at least one compatible model, respectively. The necessary and possible assignments are computed through Mixed-Integer Linear Programming (MILP).

Using the proposed approach, the DM can initially introduce only a few representative reference alternatives that she considers appropriate to be assigned to some classes. Then, we allow the progressive incorporation of preference information in the context of an outranking model assumed by Electre Tri-C, which makes it easier to associate each assignment example individually with changes in the necessary and possible assignments. Analysis of these results may stimulate the DM to interactively specify possibly imprecise assignment examples. Such interaction can be particularly useful for constructive preference learning [22]. Let us note that in the recent Electre Tri-nC method [26] each class may be also defined with several characteristic profiles. Contrary to our proposal, Electre Tri-nC requires the profiles to be provided at once, and employs them within a standard aggregation procedure for computing the alternatives' final assignments.

The fourth aim of this paper is to introduce revised, simpler versions of Electre Tri-C assignment procedures, called Electre Tri-rC. The results obtained with the revised procedures differ from those of Electre Tri-C only in very specific problem instances. The new procedures have significant implications for the ROR approach because the space of compatible outranking models is now convex. Consequently, we are able to prove a no class jumps-property for the possible assignments computed with the revised procedures, leading to easier interpretation of its results. Let us emphasize that this desirable property does not hold for the possible assignments obtained with a set of Electre Tri-B models (see [27]).

We continue by introducing the outranking preference model in Section 2 and rest of the Electre Tri-C method in Section 3. Section 4 presents the disaggregation approach for Electre Tri-C, procedures for computing the necessary and possible assignments, and the reformulated assignment procedures. Section 5 introduces Electre Tri-rC, and adapts the previous formulations to the revised assignment procedures. Section 6 describes a decision aiding process for the proposed approach. Section 7 demonstrates the use of the approach by analyzing an example. Section 8 concludes.

2. The outranking preference model

We use the following notation:

- $A = \{a_1, a_2, \ldots, a_i, \ldots\}$ – a set of decision alternatives.
- $F = \{g_1, \ldots, g_j, \ldots, g_n\}$ – a consistent family of n criteria, $g_j : A \to \mathbb{R}$; we assume, without loss of generality, that all criteria are maximized, i.e. the attractiveness increases with the criterion performance increase.
- $C_1, \ldots, C_h, \ldots, C_t$ with $t \geq 2$ – a set of pre-defined completely ordered (from the worst to the best) classes so that C_{h+1} is preferred to C_h, $h = 1, \ldots, t-1$; each class is defined with a characteristic profile[1] b_h.
- $A^R = \{a_1^*, a_2^*, \ldots\}$ – a finite set of reference alternatives on which the DM accepts to express preferences. We assume that $A^R \subseteq A$.

The binary outranking relation corresponding to the statement "at least as good as" is denoted by xSy, and its negation by xS^cy. The outranking preference model applies pseudo-criteria [28] that models per-criterion attractiveness with indifference ($q_j(x)$) and preference ($p_j(x)$) thresholds defined either as affine functions or constant values so that if

$$\begin{array}{ll} |g_j(x) - g_j(y)| \leq q_j(x), & x \text{ is indifferent to } y, \text{ denoted } xI_jy, \\ g_j(x) - g_j(y) \geq p_j(x), & x \text{ is strictly preferred to } y, \text{ denoted } xP_jy, \\ q_j < g_j(x) - g_j(y) < p_j(x), & x \text{ is weakly preferred to } y, \text{ denoted } xQ_jy. \end{array} \tag{1}$$

A set of importance coefficients (weights) $w_j \geq 0$, $j = 1, \ldots, n$, is associated with the set of criteria. Without loss of generality, we assume that $\sum_{j=1}^{n} w_j = 1$. Computing the outranking relation for a pair (x, y) involves the computation of a comprehensive concordance index $c(x, y)$, which represents the strength of the coalition of criteria being in favor of xSy:

$$c(x,y) = \sum_{j=1}^{n} c_j(x,y) = \sum_{j=1}^{n} w_j \cdot \varphi_j(x,y), \tag{2}$$

where

$$\varphi_j(x,y) = \begin{cases} 0 & \text{if } g_j(y) - g_j(x) \geq p_j(x), \\ 1 & \text{if } g_j(y) - g_j(x) \leq q_j(x), \\ [p_j(x) - (g_j(y) - g_j(x))]/[p_j(x) - q_j(x)] & \text{if } q_j(x) < g_j(y) - g_j(x) < p_j(x). \end{cases} \tag{3}$$

Veto thresholds $v_j(x)$ such that $v_j(x) \geq p_j(x)$, can be used to model the effect that an alternative cannot be at least as good as another if it has too low performance in even one of the attributes, i.e., the criterion "vetoes" against the outranking. When these are used, the outranking computation needs to take into account additionally the discordance indices $d_j(x, y)$:

$$d_j(x,y) = \begin{cases} 1 & \text{if } g_j(y) - g_j(x) \geq v_j(x), \\ 0 & \text{if } g_j(y) - g_j(x) < v_j(x). \end{cases} \tag{4}$$

[1] In [13] the b_h, $h = 1, \ldots, t-1$, are called characteristic reference alternatives, but to avoid confusing them with the reference alternatives A^R for which the DM provides assignment examples, we call them characteristic profiles.

Finally, let $\sigma(x,y)$ denote the credibility of the comprehensive outranking of x over y:

$$\sigma(x,y)=c(x,y)\prod_{j=1}^{n}(1-d_j(x,y)). \tag{5}$$

Thus, $\sigma(x,y)$ is equal to $c(x,y)$ if $\forall g_j \in F$ there is no discordance (i.e., $d_j(x,y)=0$) or discordance is not taken into account. Otherwise there is at least one criterion $g_j \in F$ for which $d_j(x,y)=1 \Rightarrow \sigma(x,y)=0$. Note that $\sigma(x,y)\in[0,1]$. We assume that an outranking relation xSy holds if $\sigma(x,y)\geq\lambda\in[0.5,1]$, where λ is a majority threshold. S can be used also for representing weak (Q) and strict (P) preference, indifference ($\sim$), and incomparability (R) as follows:

$$xSy \wedge yS^cx \iff xQy \vee xPy \iff x \succ y, \quad \text{where } \succ = \{Q\cup P\},$$

$$xSy\wedge ySx \iff x\sim y,$$

$$xS^cy\wedge yS^cx \iff xRy.$$

3. Electre Tri-C

Electre Tri-C is a sorting method for decision aiding contexts where each class C_h is defined through a characteristic profile b_h [13]. Let $B=\{b_0,b_1,\ldots,b_h,\ldots,b_t,b_{t+1}\}$ denote the set of $(t+2)$ characteristic profiles. The two extreme profiles b_0 and b_{t+1} have the worst and the best evaluations on all criteria, respectively (i.e., $g_j(b_0)<g_j(a)<g_j(b_{t+1})$ for all $a\in A$ and $g_j\in F$). Moreover, $\forall g_j\in F$, $g_j(b_1)>g_j(b_0)$ and $g_j(b_{t+1})>g_j(b_t)$. Furthermore, the characteristic profiles need to adhere to the following conditions:

- b_h needs to dominate b_{h-1}, $h=1,\ldots,t+1$, i.e. for $h=1,\ldots,t-1$, it holds that

$$\forall g_j\in F,\ \ g_j(b_{h+1})\geq g_j(b_h), \quad \text{and}$$
$$\exists g_j\in F:\ \ g_j(b_{h+1})>g_j(b_h), \tag{6}$$

- b_h does not outrank b_{h+1}, $h=1,\ldots,t-1$, which follows from the assignment of any characteristic profile b_h to class C_h by the underlying assignment rules (see [13]), i.e.,

$$\sigma(b_h,b_{h+1})<\lambda, \quad h=1,\ldots,t-1. \tag{7}$$

Electre Tri-C applies the descending and ascending assignment rules to indicate the lower and upper classes, respectively, to which an alternative could be assigned for a particular outranking model.

Definition 1. The descending rule compares a successively to b_h, for $h=t+1,\ldots,0$, for a particular majority threshold λ, seeking for the first characteristic profile b_h, such that $\sigma(a,b_h)\geq\lambda$, i.e.,

(a) If $h=t$, select C_t as a possible class to assign alternative a.
(b) If $0<h<t$: if $\sigma(b_h,a)>\sigma(a,b_{h+1})$, then select C_h as a possible class to assign a; otherwise, select C_{h+1}.
(c) If $h=0$, select C_1 as a possible class to assign a.

Definition 2. The ascending rule compares a successively to b_h, for $h=1,\ldots,t+1$, seeking for the first characteristic profile b_h, such that $\sigma(b_h,a)\geq\lambda$, i.e.,

(a) If $h=1$, select C_1 as a possible class to assign alternative a.
(b) If $1<h<(t+1)$: if $\sigma(a,b_h)>\sigma(b_{h-1},a)$, then select C_h as a possible class to assign a; otherwise, select C_{h-1}.
(c) If $h=(t+1)$, select C_t as a possible class to assign a.

Example 3.1. Consider a set of eleven alternatives $A=\{a_I,a_{II},\ldots,a_{XI}\}$ and five classes $\{C_1,C_2,C_3,C_4,C_5\}$ that are defined with characteristic profiles $B=\{b_0,b_1,b_2,b_3,b_4,b_5,b_6\}$, and a particular outranking preference model S. The credibility indices of the comprehensive outranking of the alternatives over characteristic profiles $\sigma(a_i,b_h)$, and vice-versa $\sigma(b_h,a_i)$, for $a_i\in A$ and $b_h\in B$, are presented in Table 1. The chosen majority threshold λ is equal to 0.70. Table 1 contains also the relations ($\succ$, $\sim$, $\prec$, or R) for all pairs of alternatives and profiles (a_i,b_h), as well as the classes selected with the descending (D) and ascending (A) rules. These two rules are always used together to indicate a range of classes $[C_{L^S(a)},C_{R^S(a)}]$ for alternative a.

Note that the order of outcomes of the descending and ascending assignment rules may vary, i.e. with some outranking models the ascending rule may indicate an assignment to a better class than the descending rule, whereas with other models the order can be reversed.

Definition 3. An assignment $[C_{L^S(a)},C_{R^S(a)}]$ for an alternative $a\in A$ is said to be precise if $L^S(a)=R^S(a)$, and imprecise in case $L^S(a)<R^S(a)$.

4. The disaggregation approach

The disaggregation approach presented here is restricted to the inference of the weights w_j and the majority threshold λ. That is, we assume that the intra-criterion preference information in the form of preference and indifference thresholds $p_j\geq q_j\geq 0$, $j=1,\ldots,n$, and the set of characteristic profiles $B=\{b_1,\ldots,b_h,\ldots,b_t\}$ are given. In addition, we assume that no discordance occurs, i.e. $v_j=\infty$. The appendices contain alternative formulations that (i) allow the inference of preference thresholds and characteristic profiles (Appendix H), and (ii) include the veto thresholds (Appendix I).

4.1. Preference model

The set of compatible models is defined with the following set of constraints:

$$\left.\begin{array}{l}
(B1)\ \sum_{j=1}^{n} w_j=1,\\
(B2)\ \sum_{j=1}^{n} c_j(b_h,b_{h+1})+\varepsilon\leq\lambda,\quad h=1,\ldots,t-1,\\
(B3)\ 0.5\leq\lambda\leq 1.0,\\
(B4)\ 0\leq w_j\leq 1.0,\quad \text{for } j=1,\ldots,n.\\
\text{for } j=1,\ldots,n,\ \ \forall(x,y)\in A\times B,\ \ \forall(x,y)\in B\times A\\
\quad\text{and } \forall(x,y)=(b_h,b_{h+1}),\quad h=1,\ldots,t-1,\\
(B5)\ c_j(x,y)=w_j\cdot\varphi_j(x,y)
\end{array}\right\}E(BASE)$$

where ε is an arbitrary small positive value and $w_j=c_j(b_{t+1},b_0)$.

Constraint (B1) normalizes the sum of weights of all criteria. Constraint (B2) guarantees that $b_hS^cb_{h+1}$, $h=1,\ldots,t-1$. Constraint (B3) is related to the definition of the majority threshold λ. Constraint (B4) ensures that the weight of each criterion is within the interval $[0,1]$. This is necessary because the majority threshold λ describes the minimum sum of weights of criteria required to be concordant with the outranking for it to hold. Often a more restricted constraint $0\leq w_j\leq 0.5$ is appropriate as it models, together with (B3), that no single criterion should be more important than the others considered jointly. Constraint (B5) guarantees that (2) and (3) hold.

In case the DM is able to provide inter-criteria preference information, it can be represented as linear constraints for the

Table 1
Assignments for the eleven examplary alternatives with the joint use of the descending (D) and ascending (A) rules for $\lambda = 0.7$.

Alternative		b_0	b_1	b_2	b_3	b_4	b_5	b_6	D	A	$[C_{L^S(a)}, C_{R^S(a)}]$
a_{I}	$\sigma(a_I, b_h)$	1.0	1.0	1.0	0.9	0.5	0.3	0.0	C_3	C_3	$[C_3, C_3] = C_3$
	$\sigma(b_h, a_I)$	0.0	0.0	0.2	0.6	0.7	0.8	1.0			
	(a_I, b_h)	$\succ$	$\succ$	$\succ$	$\succ$	$\prec$	$\prec$	$\prec$			
a_{II}	$\sigma(a_{II}, b_h)$	1.0	1.0	1.0	0.9	0.6	0.3	0.0	C_4	C_4	$[C_4, C_4] = C_4$
	$\sigma(b_h, a_{II})$	0.0	0.0	0.2	0.5	0.7	0.8	1.0			
	(a_{II}, b_h)	$\succ$	$\succ$	$\succ$	$\succ$	$\prec$	$\prec$	$\prec$			
a_{III}	$\sigma(a_{III}, b_h)$	1.0	1.0	1.0	0.9	0.6	0.3	0.0	C_4	C_3	$[C_3, C_4]$
	$\sigma(b_h, a_{III})$	0.0	0.0	0.2	0.6	0.7	0.8	1.0			
	(a_{III}, b_h)	$\succ$	$\succ$	$\succ$	$\succ$	$\prec$	$\prec$	$\prec$			
a_{IV}	$\sigma(a_{IV}, b_h)$	1.0	1.0	0.9	0.7	0.3	0.1	0.0	C_3	C_3	$[C_3, C_3] = C_3$
	$\sigma(b_h, a_{IV})$	0.0	0.2	0.5	0.7	0.8	1.0	1.0			
	(a_{IV}, b_h)	$\succ$	$\succ$	$\succ$	$\sim$	$\prec$	$\prec$	$\prec$			
a_{V}	$\sigma(a_V, b_h)$	1.0	1.0	0.9	0.6	0.3	0.1	0.0	C_3	C_3	$[C_3, C_3] = C_3$
	$\sigma(b_h, a_V)$	0.0	0.2	0.3	0.5	0.8	1.0	1.0			
	(a_V, b_h)	$\succ$	$\succ$	$\succ$	R	$\prec$	$\prec$	$\prec$			
a_{VI}	$\sigma(a_{VI}, b_h)$	1.0	1.0	0.9	0.6	0.5	0.1	0.0	C_3	C_4	$[C_3, C_4]$
	$\sigma(b_h, a_{VI})$	0.0	0.2	0.3	0.4	0.8	1.0	1.0			
	(a_{VI}, b_h)	$\succ$	$\succ$	$\succ$	R	$\prec$	$\prec$	$\prec$			
a_{VII}	$\sigma(a_{VII}, b_h)$	1.0	1.0	0.9	0.4	0.3	0.1	0.0	C_2	C_3	$[C_2, C_3]$
	$\sigma(b_h, a_{VII})$	0.0	0.2	0.5	0.6	0.8	1.0	1.0			
	(a_{VII}, b_h)	$\succ$	$\succ$	$\succ$	R	$\prec$	$\prec$	$\prec$			
a_{VIII}	$\sigma(a_{VIII}, b_h)$	1.0	1.0	1.0	1.0	0.8	0.3	0.0	C_4	C_3	$[C_3, C_4]$
	$\sigma(b_h, a_{VIII})$	0.0	0.2	0.5	0.9	1.0	1.0	1.0			
	(a_{VIII}, b_h)	$\succ$	$\succ$	$\succ$	$\sim$	$\sim$	$\prec$	$\prec$			
a_{IX}	$\sigma(a_{IX}, b_h)$	1.0	1.0	0.9	0.6	0.5	0.3	0.0	C_3	C_4	$[C_3, C_4]$
	$\sigma(b_h, a_{IX})$	0.0	0.2	0.2	0.3	0.5	1.0	1.0			
	(a_{IX}, b_h)	$\succ$	$\succ$	$\succ$	R	R	$\prec$	$\prec$			
a_{X}	$\sigma(a_X, b_h)$	1.0	1.0	0.9	0.6	0.6	0.5	0.0	C_3	C_5	$[C_3, C_5]$
	$\sigma(b_h, a_X)$	0.0	0.2	0.2	0.3	0.4	1.0	1.0			
	(a_X, b_h)	$\succ$	$\succ$	$\succ$	R	R	$\prec$	$\prec$			
a_{XI}	$\sigma(a_{XI}, b_h)$	1.0	1.0	0.9	0.4	0.3	0.2	0.0	C_2	C_4	$[C_2, C_4]$
	$\sigma(b_h, a_{XI})$	0.0	0.2	0.5	0.5	0.6	1.0	1.0			
	(a_{XI}, b_h)	$\succ$	$\succ$	$\succ$	R	R	$\prec$	$\prec$			

weights and/or λ. For example, partial preference information can be represented as weight intervals, $[w_{j,*}, w_j^*] \Rightarrow w_j^* \geq w_j = c_j(b_{t+1}, b_0) \geq w_{j,*}$. For other weight constraint formulations, see [29].

4.2. Assignment examples

To formulate a set of conditions that reproduce the assignment $a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$, let us first study all possible relations between an alternative a and the characteristic profiles B. For each possible scenario, we are interested in:

- relation ($\succ, \sim, \prec$, or R) that holds between a and b_k, $k = 0, \ldots, t+1$ (for explanation of $\succ, \sim, \prec$, and R, see Section 2),
- comparison ($>, \geq, =, \leq$ or $<$) between $c(a, b_{k+1})$ and $c(b_k, a)$, $k = 0, \ldots, t$.

These results are sufficient for justifying the recommendation provided by the assignment procedures of Electre Tri-C. For example:

- if a compares with B as follows:
  - for $k = 0, \ldots, h-2 : a \succ b_k$ and $c(a, b_{k+1}) > c(b_k, a)$, and
  - $a \succ b_{h-1}$ and $c(a, b_h) > c(b_{h-1}, a)$ and $a \succ b_h$ and $c(a, b_{h+1}) < c(b_h, a)$ and $a \prec b_{h+1}$, and
  - for $k = h+2, \ldots, t+1 : c(a, b_k) < c(b_{k-1}, a)$ and $a \prec b_k$,

then both the ascending (A) and descending (D) rules result in the precise assignment of a to class C_h (this is denoted by $[C_h^{A,D}]$). The descending (ascending) rule indicates C_h, because b_h is the best profile outranked by a (aSb_h) (b_{h+1} is the worst profile outranking a ($b_{h+1}Sa$)), and, moreover, $c(b_h, a) > c(a, b_{h+1})$. This scenario is exemplified in Table 1 by the comparison of a_I with the characteristic profiles $\{b_0, \ldots, b_6\}$ for the case $h=3$ and $t=5$.

- if a compares with B as follows:
  - for $k = 0, \ldots, h-2 : a \succ b_k$ and $c(a, b_{k+1}) > c(b_k, a)$, and
  - $a \succ b_{h-1}$ and $c(a, b_h) > c(b_{h-1}, a)$ and
  - for $k = h, \ldots, s$, with $h < s : a \sim b_k$, and
  - $c(a, b_{s+1}) < c(b_s, a)$ and $a \prec b_{s+1}$, and
  - for $k = s+2, \ldots, t+1 : c(a, b_k) < c(b_{k-1}, a)$, and $a \prec b_k$,

then the ascending (A) and descending (D) rules indicate, respectively, C_h and C_s, which results in the imprecise assignment $[C_h, C_s]$ (this is denoted by $[C_h^A, C_s^D]$). In particular, the descending (ascending) rule indicates C_s (C_h), because b_s is the best profile outranked by a (aSb_s) (b_h is the worst profile outranking a ($b_h S a$)), and, moreover, $c(b_s, a) > c(a, b_{s+1})$ ($c(a, b_h) > c(b_{h-1}, a)$). This scenario is exemplified in Table 1 by the comparison of a_{VIII} with characteristic profiles $\{b_0, \ldots, b_6\}$ for the case $h=3$, $s=4$, and $t=5$.

In Tables 2 and 3, we depict all possible relations between a and the characteristic profiles b_k, $k=0, \ldots, t+1$. The previous examples correspond to, respectively, columns I in Table 2 and VIII in Table 3. Each table column, denoted by $\{\text{I}, \text{II}, \ldots, \text{XI}\}$, is further exemplified in Table 1 by the comparison of respective a_i, $i \in \{\text{I}, \text{II}, \ldots, \text{XI}\}$ with b_k, $k=0, \ldots, 6$. For all table columns, the example provided in Table 1 assumes $h=3$, while for the scenarios depicted in columns $\{\text{VIII}, \ldots, \text{XI}\}$ in Table 3, we additionally assume $s=4$.

Discussion and detailed justification of the assignments is provided in Appendix A. The general idea underlying the assignment rules of Electre Tri-C is summarized as follows:

- the descending rule identifies the best characteristic profile b_{h_D}, such that aSb_{h_D}, and then compares $c(b_{h_D}, a)$ with $c(a, b_{h_D+1})$ to choose between C_{h_D} and C_{h_D+1};
- the ascending rule identifies the worst characteristic profile b_{h_A}, such that $b_{h_A} S a$, and then compares $c(a, b_{h_A})$ with $c(b_{h_A-1}, a)$ to choose between C_{h_A} and C_{h_A-1}.

In general, when comparing an alternative $a \in A$ with the characteristic profiles, one of the following situations occurs:

- a is neither indifferent nor incomparable to $b_h \in B$; then, the same class or two consecutive classes are indicated by both assignment rules; although $h_D = h_A - 1$, comparisons of the concordance indices $c(b_{h_D}, a) = c(b_{h_A-1}, a)$ and $c(a, b_{h_D+1}) = c(a, b_{h_A})$ with both rules imply that the class indicated by the ascending rule is not better than the class indicated by the descending one;
- a is indifferent to at least one characteristic profile $b_h, \ldots, b_s \in B$, $h \le s$ (in case $h < s$, profiles $b_h, \ldots, b_s$ define a subset of consecutive classes); then C_h is indicated by the ascending rule, whereas C_s is indicated by the descending rule; since $h \le s$, the class indicated by the ascending rule is not better than the class indicated by the descending one;
- a is incomparable to at least one characteristic profile $b_h, \ldots, b_s \in B$, $h \le s$ (in case $h < s$, profiles $b_h, \ldots, b_s$ define a subset of consecutive classes); then, either C_{h-1} or C_h is indicated by the descending rule, whereas C_s or C_{s+1} is indicated by the ascending rule (note, however, that C_{h-1} and C_{s+1} cannot be indicated jointly); since $h \le s$, the class indicated by the descending rule is not better than the class indicated by the ascending one.

Analysis of these assignments can be summarized with the following four remarks:

Remark 4.1. The assignment can be precise (Table 2, columns I, II, IV, V) or imprecise (Table 2, columns III, VI, VII, and Table 3, columns VIII–XI).

Remark 4.2. If there are only preference relations between a and B (Table 2, columns I–III), or a is indifferent to at least one characteristic profile (Table 2, column IV, and Table 3, column VIII), the class indicated by the descending rule is at least as good as the class indicated by the ascending rule.

Remark 4.3. If a is incomparable to at least one characteristic profile (Table 2, columns V–VII, and Table 3, columns IX–XI), the class indicated by the ascending rule is at least as good as the class indicated by the descending rule.

Remark 4.4. Let $C_{L^S(a)}$ and $C_{R^S(a)}$ be, respectively, the worst and the best classes to which a is assigned to by the outranking model S. When analyzing different scenarios, one can observe all types of relations ($\succ$, $\sim$, $\prec$, and R) between a and b_j, $k = L^S(a), \ldots, R^S(a)$. For example, $a \succ b_{L^S(a)}$ (Table 2, column I), or $a \sim b_{L^S(a)}$ (Table 2, column IV), or $a \prec b_{L^S(a)}$ (Table 2, column II), or $aRb_{L^S(a)}$ (Table 2, column V) (the same examples hold for the comparison of a with $b^S_R(a)$).

The variety of possible relations between a and b_k, $k = L^S(a), \ldots, R^S(a)$ means that the conditions for reproducing the assignment example $a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$ cannot be formulated solely by comparing a^* with the characteristic profiles b_k, $k = L^{DM}(a^*), \ldots, R^{DM}(a^*)$. Instead, we need to guarantee that a^* is

Table 3
Assignments with the joint use of the ascending (A) and descending (D) rules for the cases where the alternative a is indifferent ($\sim$) or incomparable (R) to at least two characteristic profiles $b_h, \ldots, b_s \in B$, $h < s$.

Profile index	Result	VIII $[C^A_h, C^D_s]$	IX $[C^D_h, C^A_s]$	X $[C^D_h, C^A_{s+1}]$	XI $[C^D_{h-1}, C^A_s]$
$k=0, \ldots, h-2$	(a, b_k)	$\succ$	$\succ$	$\succ$	$\succ$
	$c(a, b_{k+1})$? $c(b_k, a)$	$>$	$>$	$>$	$>$
	(a, b_{h-1})	$\succ$	$\succ$	$\succ$	$\succ$
	$c(a, b_h)$? $c(b_{h-1}, a)$	$>$	$\ge$	$\ge$	$<$
$k=h, \ldots, s$	(a, b_k)	$\sim$	R	R	R
	$c(a, b_{s+1})$? $c(b_s, a)$	$<$	$\le$	$>$	$\le$
	(a, b_{s+1})	$\prec$	$\prec$	$\prec$	$\prec$
$k=s+2, \ldots, t+1$	$c(a, b_k)$? $c(b_{k-1}, a)$	$<$	$<$	$<$	$<$
	(a, b_k)	$\prec$	$\prec$	$\prec$	$\prec$

Table 2
Assignments with the joint use of the ascending (A) and descending (D) rules for the cases where there are only preference relations ($\sim$ and R are empty) or the alternative a is indifferent ($\sim$) or incomparable (R) to only a single characteristic profile $b_h \in B$.

Profile index	Result	I $[C^{A,D}_h]$	II $[C^{A,D}_{h+1}]$	III $[C^A_h, C^D_{h+1}]$	IV $[C^{A,D}_h]$	V $[C^{A,D}_h]$	VI $[C^D_h, C^A_{h+1}]$	VII $[C^D_{h-1}, C^A_h]$
$k=0, \ldots, h-2$	(a, b_k)	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$
	$c(a, b_{k+1})$? $c(b_k, a)$	$>$	$>$	$>$	$>$	$>$	$>$	$>$
	(a, b_{h-1})	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$	$\succ$
	$c(a, b_h)$? $c(b_{h-1}, a)$	$>$	$>$	$>$	$>$	$\ge$	$\ge$	$<$
	(a, b_h)	$\succ$	$\succ$	$\succ$	$\sim$	R	R	R
	$c(a, b_{h+1})$? $c(b_h, a)$	$<$	$>$	$=$	$<$	$\le$	$>$	$\le$
	(a, b_{h+1})	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$
$k=h+2, \ldots, t+1$	$c(a, b_k)$? $c(b_{k-1}, a)$	$<$	$<$	$<$	$<$	$<$	$<$	$<$
	(a, b_k)	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$	$\prec$

assigned neither to a class worse than $C_{L^{DM}(a^*)}$ nor to a class better than $C_{R^{DM}(a^*)}$, which requires referring to the characteristic profiles of classes outside the desired interval of classes $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$.

4.3. Inference programs

As noted earlier in Remark 4.4, in order to guarantee that a^* will be assigned to the desired range of classes $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$, we need to ensure that a^* is not assigned to a class worse than $C_{L^{DM}(a^*)}$ nor to a class better than $C_{R^{DM}(a^*)}$. The conditions that prevent a^* from being assigned to the class $C_{L^{DM}(a^*)-1}$ or worse are the following:

- $c(a^*, b_{L^{DM}(a^*)-1}) \geq \lambda$ and $c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*)$ when the descending rule indicates the worst class;
- $c(b_{L^{DM}(a^*)-1}, a^*) < \lambda$ and $c(a^*, b_{L^{DM}(a^*)}) > c(b_{L^{DM}(a^*)-1}, a^*)$ when the ascending rule indicates the worst class.

Consequently, $c(a^*, b_{L^{DM}(a^*)-1}) \geq \lambda$ and $c(b_{L^{DM}(a^*)-1}, a^*) < \lambda$ are indispensable, and, thus, a^* needs to be preferred to $b_{L^{DM}(a^*)-1}$ ($a^* \succ b_{L^{DM}(a^*)-1}$). Moreover, $c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*)$ is required in case $aRb_{L^{DM}(a^*)}$ (see Remark 4.3); otherwise, the latter condition can be relaxed to a strict inequality $c(a^*, b_{L^{DM}(a^*)}) > c(b_{L^{DM}(a^*)-1}, a^*)$ (see Remark 4.2). These considerations lead to Lemma 1.

Lemma 1. *$\forall a^* \in A^R$, the set of conditions (8) prevents a^* from being assigned to a class $C_{L^{DM}(a^*)-1}$ or worse for $L^{DM}(a^*) > 1$.*

$$a^* \succ b_{L^{DM}(a^*)-1} \text{ and } \{c(a^*, b_{L^{DM}(a^*)}) > c(b_{L^{DM}(a^*)-1}, a^*) \text{ or } [c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*) \text{ and } a^*Rb_{L^{DM}(a^*)}]\}. \quad (8)$$

In terms of MILP formulation, (8) corresponds to $EL(a^*, L^{DM}(a^*))$:

$$\left.\begin{array}{l}
(EL1)\ c(a^*, b_{L^{DM}(a^*)-1}) \geq \lambda, \\
(EL2)\ c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon \leq \lambda, \\
(EL3)\ M \cdot v_L + c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon, \\
(EL4_1)\ M \cdot (1 - v_L) + c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*), \\
(EL4_2)\ M \cdot (1 - v_L) + \lambda \geq c(a^*, b_{L^{DM}(a^*)}) + \varepsilon, \\
(EL4_3)\ M \cdot (1 - v_L) + \lambda \geq c(b_{L^{DM}(a^*)}, a^*) + \varepsilon, \\
(EL5)\ v_L \in \{0, 1\}.
\end{array}\right\} EL(a^*, L^{DM}(a^*))$$

where $M > 1$ □.

Proof. In Appendix B.1.

The conditions that prevent a^* from being assigned to a class $C_{R^{DM}(a^*)+1}$ or better are the following:

- $c(a^*, b_{R^{DM}(a^*)+1}) < \lambda$ and $c(b_{R^{DM}(a^*)}, a^*) > c(a^*, b_{R^{DM}(a^*)+1})$ when the descending rule indicates the best class;
- $c(b_{R^{DM}(a^*)+1}, a^*) \geq \lambda$ and $c(b_{R^{DM}(a^*)}, a^*) \geq c(a^*, b_{R^{DM}(a^*)+1})$ when the ascending rule indicates the best class.

Consequently, $c(b_{R^{DM}(a^*)+1}, a^*) \geq \lambda$ and $c(a^*, b_{R^{DM}(a^*)+1}) < \lambda$ are indispensable, and, thus, $b_{R^{DM}(a^*)+1}$ needs to be preferred to a^* ($b_{R^{DM}(a^*)+1} \succ a^*$). Moreover, $c(b_{R^{DM}(a^*)}, a^*) \geq c(a^*, b_{R^{DM}(a^*)+1})$ is required in case $aRb_{R^{DM}(a^*)}$ (see Remark 4.3); otherwise, the latter condition can be relaxed to a strict inequality $c(b_{R^{DM}(a^*)}, a^*) > c(a^*, b_{R^{DM}(a^*)+1})$ (see Remark 4.2). These considerations lead to Lemma 2.

Lemma 2. *$\forall a^* \in A^R$, the set of (9) prevents a^* from being assigned to a class $C_{R^{DM}(a^*)+1}$ or better for $R^{DM}(a^*) < t$:*

$$b_{R^{DM}(a^*)+1} \succ a^* \text{ and } \{c(b_{R^{DM}(a^*)}, a^*) > c(a^*, b_{R^{DM}(a^*)+1}) \text{ or } [c(b_{R^{DM}(a^*)}, a^*) \geq c(a^*, b_{R^{DM}(a^*)+1}) \text{ and } a^*Rb_{R^{DM}(a^*)}]\}. \quad (9)$$

In terms of MILP formulation, (9) corresponds to $EU(a^*, R^{DM}(a^*))$.

$$\left.\begin{array}{l}
(EU1)\ c(b_{R^{DM}(a^*)+1}, a^*) \geq \lambda, \\
(EU2)\ c(a^*, b_{R^{DM}(a^*)+1}) + \varepsilon \leq \lambda, \\
(EU3)\ M \cdot v_R + c(b_{R^{DM}(a^*)}, a^*) \geq c(a^*, b_{R^{DM}(a^*)+1}) + \varepsilon, \\
(EU4_1)\ M \cdot (1 - v_R) + c(b_{R^{DM}(a^*)}, a^*) \geq c(a^*, b_{R^{DM}(a^*)+1}), \\
(EU4_2)\ M \cdot (1 - v_R) + \lambda \geq c(a^*, b_{R^{DM}(a^*)}) + \varepsilon, \\
(EU4_3)\ M \cdot (1 - v_R) + \lambda \geq c(b_{R^{DM}(a^*)}, a^*) + \varepsilon, \\
(EU5)\ v_R \in \{0, 1\}.
\end{array}\right\} EU(a^*, R^{DM}(a^*))$$

□

Proof. In Appendix B.2.

The relations corresponding to the conditions of the worst and best desired classes (i.e. (8) and (9), respectively) can be identified in Tables 2 and 3. For all possible scenarios (i.e., all table columns), when a is assigned to an interval of classes $[C_{L^S(a)}, C_{R^S(a)}]$, $b_{L^S(a)-1}$ ($b_{R^S(a)+1}$) is the best (worst) characteristic profile satisfying (8) ((9)), thus preventing a from being assigned to class worse than $C_{L^S(a)}$ (better than $C_{R^S(a)}$).

Theorem 1. *$\forall a^* \in A^R$, the constraint set given below guarantees the assignment of a^* to the range of classes $[C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$.*

$$\left.\begin{array}{ll}
EL(a^*, L^{DM}(a^*)), & \text{if } L^{DM}(a^*) > 1, \\
EU(a^*, R^{DM}(a^*)) & \text{if } R^{DM}(a^*) < t.
\end{array}\right\} E(a^* \to [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}])$$

Proof. The first condition is Lemma 1 and the second one is Lemma 2. □

The set of outranking models $\mathcal{S}$ compatible with the preference information provided by the DM is defined with the following set of constraints $E(A^R)$:

$$\left.\begin{array}{l}
E(BASE), \\
E(a^* \to [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]) \text{ for all } a^* \in A^R.
\end{array}\right\} E(A^R)$$

The set $\mathcal{S}$ is not empty if $E(A^R)$ is feasible and $\varepsilon^* = \max \varepsilon$, s.t. $E(A^R)$, is greater than zero. For inconsistency analysis, see Appendix C.

4.4. Possible and necessary assignments

Definition 4. Given a set of outranking models $\mathcal{S}$ compatible with the preference information provided by the DM, the possible assignment $C_P(a)$ is defined as the set of indices of classes C_h for which there exists at least one compatible outranking model assigning a to C_h, and the necessary assignment $C_N(a)$ as the set of indices of classes C_h for which all compatible outranking models assign a to C_h. That is, the necessary and possible assignments are:

$$C_P(a) = \{h \in H : \exists S \in \mathcal{S},\ L^S(a) \leq h \leq R^S(a)\},$$
$$C_N(a) = \{h \in H : \forall S \in \mathcal{S},\ L^S(a) \leq h \leq R^S(a)\}.$$

The notion of possible assignments has been used before in the context of value- [30–33], outranking- [27,34], and rule-based approaches [45], whereas the concept of necessary assignment has been used so far only in value- and rule-based sorting approaches [30,31,45].

4.4.1. Possible assignment

Alternative a is possibly assigned to class C_h if the following two conditions are satisfied for a compatible outranking model $S \in \mathcal{S}$:

- the worse class assignment $C_{L^S(a)}$ is not better than C_h, i.e. $L^S(a) \le h \Leftrightarrow \neg(L^S(a) \ge h+1)$;
- the best class assignment $C_{R^S(a)}$ is not worse than C_h, i.e. $R^S(a) \ge h \Leftrightarrow \neg(R^S(a) \le h-1)$.

Note that a is assigned to a class at least as good as C_{h+1} (or not better than C_{h-1}), if the set of conditions $EL(a,h+1)$ (or $EU(a,h-1)$) can be reproduced within $E(A^R)$. Both $EL(a,h+1)$ and $EU(a,h-1)$ have the following form:

$$\{c_1 \text{ and } c_2 \text{ and } [c_3 \text{ or } (c_{41} \text{ and } c_{42} \text{ and } c_{43})]\}. \tag{10}$$

To show that $EL(a,h+1)$ and $EU(a,h-1)$ do not hold for any compatible outranking model, it is sufficient to prove that their negations are satisfiable within $E(A^R)$. That is,

$$\text{not}\{c_1 \text{ and } c_2 \text{ and } [c_3 \text{ or } (c_{41} \text{ and } c_{42} \text{ and } c_{43})]\} \Leftrightarrow$$

$$\{\text{not}(c_1) \text{ or } \text{not}(c_2) \text{ or } [\text{not}(c_3) \text{ and } (\text{not}(c_{41}) \text{ or } \text{not}(c_{42}) \text{ or } \text{not}(c_{43})]\}. \tag{11}$$

Let us denote the negations of $EL(a,h+1)$ and $EU(a,h-1)$ as the constraint sets $ENL(a,h)$ and $ENU(a,h)$, respectively. Then, the possible assignment of $a \in A$ can be computed by considering Theorem 2 for each $h \in H$.

Theorem 2. *$\forall a \in A, \forall h \in H, \exists S \in \mathcal{S} : L^S(a) \le h \le R^S(a)$, i.e. $a \to^P C_h$ iff the constraint set given below is feasible and $\varepsilon^* = \max \varepsilon$ s.t. $E(a \to^P C_h) > 0$:*

$$\left.\begin{array}{l}
\textit{if } h<t: \\
\left.\begin{array}{l}
(LP1)\ M\cdot(v_{L1}-1)+c(a,b_h)+\varepsilon \le \lambda, \\
(LP2)\ M\cdot(1-v_{L2})+c(b_h,a) \ge \lambda, \\
(LP3)\ M\cdot(1-v_{L3})+c(b_h,a) \ge c(a,b_{h+1}), \\
(LP4)\ M\cdot(1-v_{L41})+c(b_h,a) \ge c(a,b_{h+1})+\varepsilon, \\
(LP5)\ M\cdot(1-v_{L42})+c(a,b_{h+1}) \ge \lambda, \\
(LP6)\ M\cdot(1-v_{L43})+c(b_{h+1},a) \ge \lambda, \\
(LP7)\ v_{L1}+v_{L2}+v_{L3}=1, \\
(LP8)\ v_{L41}+v_{L42}+v_{L43}=v_{L3}, \\
(LP9)\ v_{L1},v_{L2},v_{L3},v_{L41},v_{L42},v_{L43} \in \{0,1\},
\end{array}\right\} ENL(a,h) \\
\textit{if } h>1: \\
\left.\begin{array}{l}
(UP1)\ M\cdot(v_{R1}-1)+c(b_h,a)+\varepsilon \le \lambda \\
(UP2)\ M\cdot(1-v_{R2})+c(a,b_h) \ge \lambda, \\
(UP3)\ M\cdot(1-v_{R3})+c(a,b_h) \ge c(b_{h-1},a), \\
(UP4)\ M\cdot(1-v_{R41})+c(a,b_h) \ge c(b_{h-1},a)+\varepsilon, \\
(UP5)\ M\cdot(1-v_{R42})+c(a,b_{h-1}) \ge \lambda, \\
(UP6)\ M\cdot(1-v_{R43})+c(b_{h-1},a) \ge \lambda, \\
(UP7)\ v_{R1}+v_{R2}+v_{R3}=1, \\
(UP8)\ v_{R41}+v_{R42}+v_{R43}=v_{R3}, \\
(UP9)\ v_{R1},v_{R2},v_{R3},v_{R41},v_{R42},v_{R43} \in \{0,1\},
\end{array}\right\} ENU(a,h) \\
E(A^R).
\end{array}\right\} E(a \to^P C_h) \tag{12}$$

□

Proof. In Appendix D.1.

The set $\mathcal{S}$ is non-convex as the space of compatible models is defined with a logical OR combination of linear inequalities in $E(A^R)$. This implies that there may be jumps in the possible assignment, i.e. if $\exists S', S'' \in \mathcal{S} : R^{S'}(a) \le h$ and $L^{S''}(a) \ge h+\delta^h$ with $\delta^h \ge 2$, there may exist $\delta \in \{1,\ldots,\delta^h-1\}$ for which there is no $S^* \in \mathcal{S}$ such that $L^{S^*}(a) \le h+\delta \le R^{S^*}(a)$. Nevertheless, in Section 5, we further revise the assignment rules of Electre Tri-C so that the set $\mathcal{S}$ is convex and the resulting possible assignment has no class jumps.

4.4.2. Necessary assignment

Alternative a is necessarily assigned to class C_h, $h \in H$ if the following two conditions are satisfied:

- a is assigned to a class worse than C_{h+1} with all compatible outranking models, i.e. $L^S(a) < h+1$, $\forall S \in \mathcal{S}$;
- a is assigned to class better than C_{h-1} with all compatible outranking models, i.e. $R^S(a) > h-1$, $\forall S \in \mathcal{S}$.

Note that a is assigned to a class at least as good as C_{h+1} (or not better than C_{h-1}), if the set of conditions $EL(a,h+1)$ (or $EU(a,h-1)$) can be reproduced within $E(A^R)$. If none of these is possible, then a is assigned to C_h with all $S \in \mathcal{S}$. Thus, the necessary assignment for $a \in A$ can be computed by considering Theorem 3 for each $h \in H$.

Theorem 3. *$\forall a \in A, \forall h \in H, \forall S \in \mathcal{S}, L^S(a) \le h \le R^S(a)$, i.e. $a \to^N C_h$, iff: the constraint set given below is infeasible or $\varepsilon^* =$ max ε s.t. $E(a \to^P C_{\ge h+1}) \le 0$ for $h<t$.*

$$\left.\begin{array}{l} EL(a,h+1), \\ E(A^R), \end{array}\right\} E(a \to^P C_{\ge h+1}) \tag{13}$$

and the constraint set given below is infeasible or $\varepsilon^* =$ max ε s.t. $E(a \to^P C_{\le h-1}) \le 0$ for $h>1$:

$$\left.\begin{array}{l} EU(a,h-1), \\ E(A^R). \end{array}\right\} E(a \to^P C_{\le h-1}) \tag{14}$$

□

Proof. In Appendix D.2.

Theorem 4 (*No class jumps in the necessary assignment*). *$\forall a \in A$, $C_N(a)$ is either empty or formed by the set of contiguous classes between $C_{L^N(a)}$ and $C_{R^N(a)}$ being, respectively, the worst and the best class to which a is assigned by all compatible outranking models.*

Proof. In Appendix E □.

4.5. Reformulation of the Electre Tri-C assignment procedures

The results in the previous subsections allow reformulating the assignment rules of Electre Tri-C. To indicate the worst class in which a can be assigned to, compare a successively to b_h, for $h = t-1,\ldots,0$, seeking the first characteristic profile b_h such that

$$a \succ b_h \text{ and } \{c(a,b_{h+1}) > c(b_h,a) \text{ or } [c(a,b_{h+1}) \ge c(b_h,a) \text{ and } aRb_{h+1}]\}. \tag{15}$$

Select C_{h+1}.

Proof. In Appendix F.1 □.

To indicate the best class in which a can be assigned to, compare a successively to b_h, for $h = 2,\ldots,t+1$, seeking the first characteristic profile b_h such that

$$b_h \succ a \text{ and } \{c(b_{h-1},a) > c(a,b_h) \text{ or } [c(b_{h-1},a) \ge c(b_h,a) \text{ and } aRb_{h-1}]\}. \tag{16}$$

Select C_{h-1}.

Proof. In Appendix F.2 □.

These rules unambiguously indicate the worst and best classes for the possible assignment of $a \in A$. That is, irrespective of how a compares with B (i.e., no matter if there are only preference relations between a and B, or a is indifferent or incomparable with some characteristic profiles), we can identify the extreme assignments using univocal and precise conditions. On the contrary, as proven in Section 4.2, the order of classes indicated by the original Electre Tri-C rules may vary, so one needs to use both of them in order to know the worst or the best class an alternative can be assigned to.

Moreover, translation of the reformulated procedures to explanations formulated in the natural language is straightforward as, for example, in case C_h is the worst (best) class of a, it can be justified with a being clearly better (worse) than the characteristic profile b_{h-1} ($a \succ b_{h-1}$) (b_{h+1} ($b_{h+1} \succ a$)) and there existing sufficiently strong arguments in support of a (b_h) being at least as good as b_h (a) (e.g., $c(a,b_h) > c(b_{h-1},a)$ ($c(b_h,a) > c(a,b_{h+1})$)). Such explanations cannot be derived from the original assignment rules of Electre Tri-C. Note that the topic of explaining the results of decision aiding methods has recently motivated various studies (see e.g. [35–37]).

Finally, comprehensibility of the reformulated procedures may be enhanced by drawing analogies with the well-known assignment rules of Electre Tri-B. First, the underlying logic can be explained in a similar way: in Electre Tri-B, if the pessimistic (optimistic) rule indicates C_h for the assignment of a, it can be justified with a outranking the boundary profile b_h (a being clearly worse than b_{h+1}). Second, analogously to the reformulated Electre Tri-C rules, the order of classes indicated by the pessimistic and optimistic rules is unambiguous, i.e., the previous one always indicates a class which is not better than the class indicated by the latter one. Nevertheless, there are some important differences between the two methods. On one hand, the assignment rules of Electre Tri-B are used separately (i.e., only one of them should be applied), and they indicate precise class assignments, whereas the reformulated rules of Electre Tri-C need to be used jointly for indicating a possibly imprecise interval of classes. Moreover, there is a fundamental difference in the interpretability of the characteristic profiles of Electre Tri-C and the boundary profiles of Electre Tri-B. A detailed comparison of the assumptions, requirements, and strengths of the two methods is provided in [13].

5. Electre Tri-rC: Electre Tri-C with revised assignment rules

In this section, we further revise Electre Tri-C by removing the logical OR conditions from the assignment rules given in Section 4.5. This leads to a simpler interpretation of the assignment procedures and the formulation of less complex mathematical programs. It also implies that the space of compatible outranking models (weights and λ) is convex and thus the possible assignment has no class jumps. In the proposed reformulation, (15) is replaced by

$$a \succ b_h \text{ and } c(a,b_{h+1}) > c(b_h,a), \tag{17}$$

whereas (16) is replaced by

$$b_h \succ a \quad \text{and} \quad c(b_{h-1},a) > c(a,b_h). \tag{18}$$

Remark 5.1. When compared to the original assignment rules of Electre Tri-C, the new assignment procedures would indicate:

(i) C_{h-1} rather than C_h in case $a \succ b_{h-1}$ and $c(a,b_h) = c(b_{h-1},a)$ and aRb_h;
(ii) C_{h+1} rather than C_h in case $b_{h+1} \succ a$ and $c(b_h,a) = c(a,b_{h+1})$ and aRb_h.

Thus, in case aRb_h and $a \succ b_{h-1}$ or $a \prec b_{h+1}$, the conditions for indicating class C_h instead of, respectively, C_{h-1} or C_{h+1}, are slightly more demanding than in Electre Tri-C (a weak inequality is replaced by the strict one). The recommendations provided for all other cases are equal.

Example 5.1. For all eleven alternatives provided in Table 1, the assignments indicated by the revised rules of Electre Tri-rC and the original rules of Electre Tri-C are the same. The differences in the indicated recommendations are illustrated in Table 4. Precisely, cases (i) and (ii) described in Remark 5.1 are exemplified by the assignments of, respectively, a_{XII} and a_{XIII} (we assume $h = 3$).

Let us now adapt the mathematical formulations from Section 4 to the revised assignment rules. We discuss only changes in the constraints, while the conclusions drawn from the solutions of the mathematical programs remain the same. In particular, the inference programs now can be modeled with Linear Programming (LP) rather than MILP. Precisely, $EL(a^*, L^{DM}(a^*))$ becomes

$$\left.\begin{array}{l} (EL1')\ c(a^*, b_{L^{DM}(a^*)-1}) \geq \lambda, \\ (EL2')\ c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon \leq \lambda, \\ (EL3')\ c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon, \end{array}\right\} EL'(a^*, L^{DM}(a^*))$$

and $EU(a^*, R^{DM}(a^*))$ becomes

$$\left.\begin{array}{l} (EU1')\ c(b_{R^{DM}(a^*)+1}, a^*) \geq \lambda, \\ (EU2')\ c(a^*, b_{R^{DM}(a^*)+1}) + \varepsilon \leq \lambda, \\ (EU3')\ c(b_{R^{DM}(a^*)}, a^*) \geq c(a^*, b_{R^{DM}(a^*)+1}) + \varepsilon. \end{array}\right\} EU'(a^*, R^{DM}(a^*))$$

Then, these constraints are used to define the set of compatible outranking models $E'(A^R)$, which are used for computing the possible assignment $E(a \rightarrow^P C_h)$:

$$\left.\begin{array}{l} \text{if } h < t: \\ (LP1), (LP2), (LP3), (LP7), \} ENL'(a,h) \\ \text{if } h > 1: \\ (UP1), (UP2), (UP3), (UP7), \} ENU'(a,h) \\ E'(A^R). \end{array}\right\} E'(a \rightarrow^P C_h) \tag{19}$$

Table 4
Assignment by the use of the original rules of Electre Tri-C and the revised rules of Electre Tri-rC for the two exemplary alternatives for $\lambda = 0.7$.

Alternative	Credibility/relation	b_0	b_1	b_2	b_3	b_4	b_5	b_6	Assignment by: Original rules	Assignment by: Revised rules
a_{XII}	$\sigma(a_{XII}, b_h)$	1.0	1.0	0.8	0.5	0.3	0.0	0.0	$[C_3, C_3]$	$[C_2, C_3]$
	$\sigma(b_h, a_{XII})$	0.0	0.2	0.5	0.6	0.8	1.0	1.0		
	(a_{XII}, b_h)	$\succ$	$\succ$	$\succ$	R	$\prec$	$\prec$	$\prec$		
a_{XIII}	$\sigma(a_{XIII}, b_h)$	1.0	1.0	0.8	0.6	0.5	0.0	0.0	$[C_3, C_3]$	$[C_3, C_4]$
	$\sigma(b_h, a_{XIII})$	0.0	0.0	0.2	0.5	0.8	1.0	1.0		
	(a_{XIII}, b_h)	$\succ$	$\succ$	$\succ$	R	$\prec$	$\prec$	$\prec$		

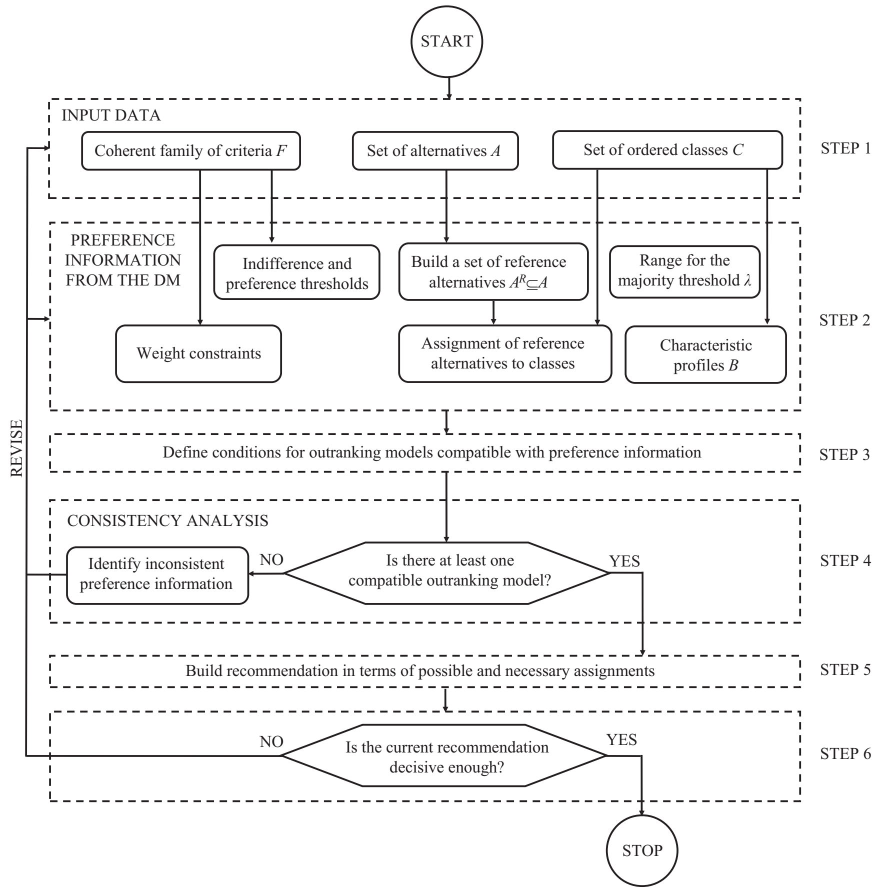

Fig. 1. Interactive decision aiding process for the proposed disaggregation approach.

For the revised assignment procedures, there are no class jumps in the possible assignment.

Theorem 5 (*No class jumps in the possible assignment*). *Denote by* $\eta(a,b_h,S)=c(a,b_h)-\lambda=\sum_{j=1}^{n}c_j(a,b_h)-\lambda$. *Let* $\mathcal{S}$ *be a convex set of compatible outranking models (weights and* λ*), and therefore,*

$$\forall S^{\mathrm{I}},S^{\mathrm{II}}\in\mathcal{S},\ \text{and}\ \alpha,\beta\in[0,1],\ S^{\mathrm{III}}\in\mathcal{S}\ \text{if}\ \eta(a,b_h,S^{\mathrm{III}}) = \alpha\eta(a,b_h,S^{\mathrm{I}})+(1-\alpha)\eta(a,b_h,S^{\mathrm{II}})$$

$$\text{and}\ \eta(b_h,a,S^{\mathrm{III}})=\beta\eta(b_h,a,S^{\mathrm{I}})+(1-\beta)\eta(b_h,a,S^{\mathrm{II}}).$$

Now, if $\exists S',S''\in\mathcal{S}:R^{S'}(a)\le h$ *and* $L^{S''}(a)\ge h+\delta^h$ *with* $\delta^h\ge 2\Longrightarrow \forall\delta\in\{0,\ldots,\delta^h\}\ \exists S^*\in\mathcal{S}$ *such that* $L^{S^*}(a)\le h+\delta\le R^{S^*}(a)$ $\square$.

Proof. In Appendix G.

When computing the necessary assignment, the constraint sets given below should be used instead of the ones in Theorem 3:

$$\left.\begin{array}{l} EL'(a,h+1),\\ E'(A^R),\end{array}\right\}E'(a\rightarrow^{P}C_{\ge h+1}) \tag{20}$$

$$\left.\begin{array}{l} EU'(a,h-1),\\ E'(A^R).\end{array}\right\}E'(a\rightarrow^{P}C_{\le h-1}) \tag{21}$$

There are no class jumps in the necessary assignment for the revised procedures. The proof is analogous to Theorem 4.

6. Decision aiding with the proposed approach

Sorting decisions can be aided with the proposed approach through the iterative six-step process illustrated in Fig. 1. The process begins by defining the problem; a set of alternatives A, a set of criteria F, the alternatives' evaluations on the criteria, and a set of ordered classes C. Then, in Step 2 the preference information is elicited and incorporated into the model. The DM needs to provide the characteristic profiles B for each class, and the indifference $q_j(x)$ and preference $p_j(x)$ thresholds for all criteria. Moreover, we assume that the DM provides a set of assignment examples consisting of reference alternatives $A^R\subseteq A$ and their desired assignments. The DM may provide linear constraints for the weights (e.g., g_j is more important than g_k, represented as $w_j>w_k$) or for the majority threshold λ.

Step 3 consists of constructing the disaggregation model. Then, in Step 4, we verify whether there exists at least one instance of the outranking model compatible with the preference information. If no such instance exists, the DM is asked to revise her preference information (see Appendix C for details).

Step 5 consists of building a recommendation in terms of the necessary and possible assignments. These are verified in Step 6; if the DM is satisfied with the recommendation, the process ends. Otherwise, one should revise the input data and/or preference information. The suggested procedure is to provide new assignment examples for alternatives with multiple possible assignments and empty necessary assignments. In the same spirit, the DM may wish to make more precise the assignments of some reference alternatives already considered in the previous iteration.

7. Illustrative study: environmental risk assessment

We re-analyze a real-world risk assessment problem of zoning the watershed of Moulinet and Violettes (Low Normandy, France).

Table 5
Land zones' performances (names as in [38]) and their possible and necessary assignments to the four risk classes in the first, second, and third iterations.

Alt.	g_1	g_2	g_3	g_4	g_5	$C_P^1(a)$	$C_N^1(a)$	$C_P^2(a)$	$C_N^2(a)$	$C_P^3(a)$	$C_N^3(a)$
a_2	10.1	2	1	1	1	C_4	C_4	C_4	C_4	C_4	C_4
a_3	8.3	2	6	1	1	C_2-C_4	–	C_4	C_4	C_4	C_4
a_4	20.3	2	4	1	1	C_3-C_4	–	C_4	C_4	C_4	C_4
a_5	219.5	3	4	4	1	C_3	C_3	C_3	C_3	C_3	C_3
a_6	49.9	7	6	4	2	C_1-C_2	C_2	C_1-C_2	C_2	C_1-C_2	C_2
a_7	208.9	7	6	1	8	C_1	C_1	C_1	C_1	C_1	C_1
a_8	67.7	2	1	4	1	C_3-C_4	C_4	C_3-C_4	C_4	C_3-C_4	C_4
a_9	141.1	3	4	4	1	C_3	C_3	C_3	C_3	C_3	C_3
a_{10}	533.6	3	4	4	1	C_3	C_3	C_3	C_3	C_3	C_3
a_{11}	134.9	1	4	4	1	C_3-C_4	–	C_3	C_3	C_3	C_3
a_{12}	91.6	1	4	4	1	C_3-C_4	–	C_3	C_3	C_3	C_3
a_{13}	129.7	1	4	4	1	C_3-C_4	–	C_3	C_3	C_3	C_3
a_{14}	44.8	1	4	3	1	C_3-C_4	–	C_3-C_4	–	C_4	C_4
a_{15}	8.3	1	4	3	1	C_3-C_4	–	C_3-C_4	–	C_4	C_4
a_{16}	14.8	1	6	1	1	C_2-C_4	–	C_4	C_4	C_4	C_4
a_{17}	53.8	1	1	1	1	C_4	C_4	C_4	C_4	C_4	C_4
a_{18}	124.9	1	1	1	1	C_4	C_4	C_4	C_4	C_4	C_4
a_{19}	89.4	1	1	4	1	C_4	C_4	C_4	C_4	C_4	C_4
a_{20}	289.2	1	4	3	1	C_3-C_4	–	C_3	C_3	C_3	C_3
a_{21}	66.4	1	6	1	1	C_2-C_4	–	C_3-C_4	–	C_3-C_4	–
a_{22}	128.5	5	4	1	1	C_2-C_3	C_2	C_2-C_3	C_2	C_2-C_3	C_2
a_{24}	55.0	3	4	1	1	C_3-C_4	–	C_3-C_4	–	C_3-C_4	–
a_{25}	135.4	7	6	1	2	C_2	C_2	C_2	C_2	C_2	C_2
a_{26}	161.4	7	6	1	2	C_1-C_2	C_1	C_1-C_2	C_1	C_1-C_2	C_1
a_{27}	163.1	7	6	1	1	C_1-C_3	C_1	C_1-C_3	C_1	C_1-C_3	C_1
a_{28}	244.8	5	4	4	1	C_2	C_2	C_2	C_2	C_2	C_2
a_{29}	215.1	3	4	4	1	C_3	C_3	C_3	C_3	C_3	C_3
a_{30}	49.8	3	4	6	1	C_2-C_4	–	C_2-C_4	–	C_3	C_3
a_{31}	66.4	3	1	1	1	C_4	C_4	C_4	C_4	C_4	C_4
a_{38}	59.9	7	6	1	2	C_2	C_2	C_2	C_2	C_2	C_2
a_{42}	117.6	5	4	4	1	C_2	C_2	C_2	C_2	C_2	C_2
a_{43}	207	7	6	1	2	C_1-C_2	C_1	C_1-C_2	C_1	C_1-C_2	C_1
a_{44}	431.3	5	4	3	1	C_2	C_2	C_2	C_2	C_2	C_2
a_{45}	62	3	6	3	1	C_2-C_3	C_3	C_3	C_3	C_3	C_3
a_{59}	576.7	7	6	1	5	C_1	C_1	C_1	C_1	C_1	C_1
a_{75}	273.9	7	6	1	1	C_1-C_3	C_1	C_1-C_3	C_1	C_1-C_3	C_1
a_{76}	289.6	7	6	1	8	C_1	C_1	C_1	C_1	C_1	C_1
a_{106}	91.4	7	6	1	2	C_2	C_2	C_2	C_2	C_2	C_2
a_{107}	135.8	5	6	6	1	C_1-C_2	–	C_1-C_2	–	C_2	C_2
a_{108}	108.2	5	6	4	1	C_2	C_2	C_2	C_2	C_2	C_2

Table 6
Indifference and preference thresholds for the environmental risk assessment problem.

	g_1	g_2	g_3	g_4	g_5
$q_j(x)$	$0.01 \cdot g_1(x)$	0	0	0	0
$p_j(x)$	$0.05 \cdot g_1(x)$	1.9	1.9	1.9	2

Table 7
Characteristic profiles for the environmental risk assessment problem.

	g_1	g_2	g_3	g_4	g_5
b_1	200	7	6	6	8
b_2	150	5	4	4	5
b_3	100	3	2	3	2
b_4	50	2	1	1	1

Table 8
Assignment examples for the environmental risk assessment problem.

Class	Assigned land zones		
	1st iteration	2nd iteration	3rd iteration
C_1	a_7	–	–
C_2	a_{38}, a_{108}	–	a_{107}
C_3	a_{10}	a_{12}	a_{30}
C_4	a_{31}	a_4, a_{16}	a_{15}

The problem was originally analyzed in [38] with Electre Tri-C. We consider a set of 40 land zones evaluated on five criteria with decreasing direction of preference:

- g_1: overall slope of the land zone;
- g_2: quality of the connectivity between the land zone and the stream;
- g_3: type of embankment in the lower part of the land zone;
- g_4: nature of crops in the land zone;
- g_5: bank alteration by the cows when they drink water directly from the stream.

The zones' evaluations are presented in Table 5. The objective of the study is to indicate the most appropriate intervention for protecting the reproduction habitat of salmonidae fish in these watersheds. The interventions are designed based on the following risk classes: very high risk (C_1), high risk (C_2), intermediate risk (C_3), and low or no risk (C_4). The presented results were obtained with respect to the original assignment rules of Electre Tri-C using algorithms from Section 4.

We assume that no single criterion is more important than the others considered jointly, i.e. the constraint (B4) in $E(BASE)$ is reformulated as the more restricted one with $0 \le w_j \le 0.5$. Computation of the necessary and possible assignments was implemented via constructing MILPs that were subsequently solved with glpk. Our R implementation is freely available online.[2] The indifference and preference thresholds provided by the DM are given in Table 6, the characteristic profiles are provided in Table 7, and the exemplary assignments of the five reference zones supplied in the first iteration are given in Table 8. These are consistent and the set of compatible outranking models is not empty.

The possible and necessary assignments are presented in Table 5 (columns $C_P^1(a)$ and $C_N^1(a)$). The necessary assignment is precise and not empty for the five reference zones (a_7, a_{38}, a_{108}, a_{10}, a_{31}). Another 22 non-reference zones (e.g., a_{59}, a_{25}, a_{29}, a_{17}) are assigned precisely, i.e. to a single class with all compatible outranking models. For the remaining 13 zones, the necessary assignment is empty and the possible assignments are imprecise. However, there are also 8 zones for which the possible assignments are imprecise, although the necessary ones are non-empty. For example, with some compatible outranking models a_6 is assigned to $[C_1, C_2]$, while with the remaining ones to C_2. As a result $C_P(a_6) = [C_1 - C_2]$ and $C_N(a_6) = C_2$. Overall, there are 15 zones possibly assigned to two consecutive classes (i.e., $C_1 - C_2$ or $C_2 - C_3$ or $C_3 - C_4$) and 6 zones with a possible assignment in three classes (i.e., $C_1 - C_3$ or $C_2 - C_4$). The average difference between the extreme class indices is 0.7.

Let us suppose that considering the results of the first iteration, the DM feels confident that a_{12} should be assigned to C_3, while a_4 and a_{16} belong to C_4 (see Table 8). Comparing to the previous iteration, there are another 7 zones with a non-empty precise necessary assignment $C_N^2(a)$ (see Table 5). Also the possible assignments $C_P^2(a)$ are now more precise. For 9 zones, including 6 non-reference ones (a_{11}, a_{13}, a_{20}, a_{45}, a_3, a_{21}), $C_P^2(a)$ is more tight than $C_P^1(a)$. The average difference between the extreme class indices is now 0.43.

[2] http://github.com/tommite/pubs-code/tree/master/etricror-omega/

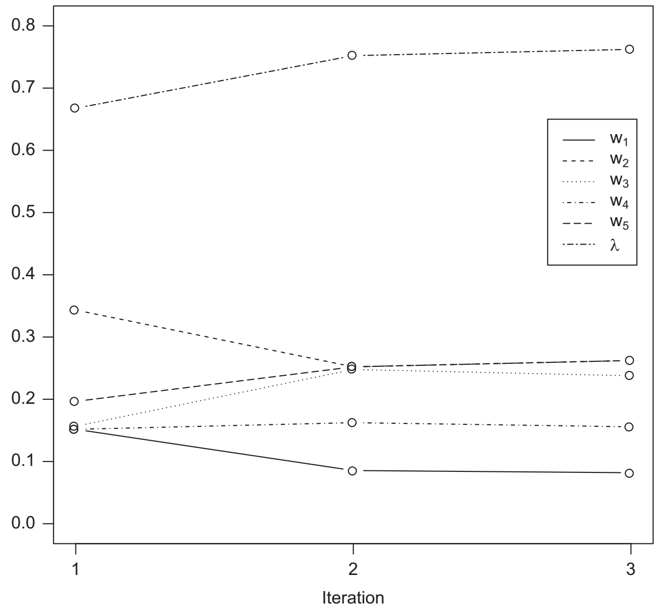

Fig. 2. Criteria importance coefficients (w_1-w_5) and majority threshold λ obtained for the selected compatible outranking models in the three iterations.

In the third iteration, the DM provides additional exemplary assignments (see Table 8). As a result, the possible assignments for 4 zones have been tightened and the necessary ones have been enriched (see Table 5, columns $C_P^3(a)$ and $C_N^3(a)$). In case the DM is not yet satisfied with the results, she may want to continue the iterative process, either by adding some new assignments of reference zones or by revising the previous judgments.

Note that in each iteration the DM can be supported by displaying a single representative compatible outranking model. Such a representative model may be obtained by maximizing ε, s.t. $E(A^R)$. Representative models selected in this way for the three iterations of the case study are presented in Fig. 2. The outranking preference model may be easier to explain to the DM through these values: for example, the representative model in the third iteration has the majority threshold $\lambda \approx 0.76$ and the criteria weights: $w_1 \approx 0.09$, $w_2 = w_5 \approx 0.26$, $w_3 \approx 0.23$, and $w_4 \approx 0.16$. As a result, when referring to this representative model, the outranking relation holds if one of the following coalitions of criteria fully supports it: $\{g_1, g_2, g_3, g_5\}$, $\{g_1, g_2, g_4, g_5\}$, or $\{g_2, g_3, g_4, g_5\}$ (e.g., in the latter case, $w_2+w_3+w_4+w_5 \approx 0.91 > 0.76$).

We also analyzed the same problem with the revised assignment rules and respective algorithms from Section 5. In all three iterations, the possible and necessary assignments (Table 5) as well as the selected representative outranking models (Fig. 2) were exactly the same as the ones obtained with the original assignment procedures of Electre Tri-C. This suggests that the impact of using the revised assignment rules on the space of compatible outranking models and the sorting recommendation is rather minor.

Our previous work [31] developed inference programs for sorting procedures applying general multi-attribute value functions, and augmented the possible relations with information about the shares of compatible models assigning alternatives to the different classes. With general value functions the conditions for inferring additional necessary relations apart from the transitivity of $\succsim$ are very specific [39], and the resulting ranges of possible assignments are often rather wide, whereas the set of necessary assignments is often empty (see e.g. [40,41]). Therefore, additional information in the form of shares of the compatible models can be very useful when applying general additive value models. Conversely, when applying sets of outranking sorting models parameterized with the intra-criterion parameters (thresholds) and the characteristic class profiles, the space of compatible models is considerably smaller. Thus, the width of the possible class assignments may decrease greatly when restricting the set of models with additional assignment examples (see Table 5). Therefore, less preference information is required for arriving at a final recommendation with the more restrictive outranking preference model than with general value functions.

8. Conclusions

We presented a new approach for multiple criteria sorting problems using characteristic profiles for defining the classes, similarly to the Electre Tri-C method. The order of the extreme classes indicated by the Electre Tri-C assignment rules vary depending on the considered outranking model. We reformulated the rules to indicate the worst and best classes for each alternative in an unambiguous way, therefore enhancing interpretability of the sorting recommendation.

We also introduced a disaggregation procedure for inferring a set of outranking models compatible with the possibly imprecise assignment examples. In this way, we avoid asking the DM directly for the model parameter values. We considered a model with unknown criteria weights and majority threshold, and additionally accounted for inferring the comparison and veto thresholds as well as the characteristic class profiles. We adapted Electre Tri-C for analyses with sets of outranking models. For each alternative, we determined the possible and necessary assignments.

Finally, we revised the assignment procedures of Electre Tri-C so, that the space of compatible outranking models $\mathcal{S}$ is convex and there are no class jumps in the possible assignment. The introduced approaches were illustrated by re-analyzing a real-world problem of sorting land zones into four classes representing different risk levels.

We envisage the following developments of the proposed approach:

- inclusion of new types of preference information, such as desired class cardinalities [42,43],
- higher variety of results, for example, assignment-based preference relations [31], and
- extensions of the method for group decision making problems.

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Appendix A. Discussion of possible relations between an alternative *a* and the characteristic profiles

There are three possible settings concerning relations between a and $B = \{b_0, \ldots, b_{t+1}\}$:

1. If there are *only preference relations between a and B* ($\sim$ and R are empty), then $\exists h \in \{0, \ldots, t\}$: $a \succ b_0$, $a \succ b_1, \ldots, a \succ b_h$, $b_{h+1} \succ a, \ldots, b_{t+1} \succ a$. According to the descending rule, the highest index ϕ such that a is preferred to b_ϕ is $\phi = h$. Thus, if $\sigma(a, b_{h+1}) < \sigma(b_h, a)$, then class C_h is selected for the assignment

of a;otherwise, C_{h+1} is selected. According to the ascending rule, the lowest index φ such that b_φ is preferred to a is $\varphi=(h+1)$. Thus, if $\sigma(a,b_{h+1})>\sigma(b_h,a)$, C_{h+1} is selected for the assignment of a;otherwise, C_h is selected. Consequently, the two rules select either the same class: C_h (in case $\sigma(a,b_{h+1})<\sigma(b_h,a)$) or C_{h+1} (in case $\sigma(a,b_{h+1})>\sigma(b_h,a)$) or the ascending rule selects class C_h and the descending rule C_{h+1} (in case $\sigma(a,b_{h+1})=\sigma(b_h,a)$). See Table 2 (columns I–III).

2. If a is *indifferent to at least one characteristic profile*, then $\exists h\in\{0,\ldots,t-1\}$, $\exists s\in\{h+1,\ldots,t\}$: $a\succ b_0$, $a\succ b_1,\ldots,a\succ b_{h-1}$, $a\sim b_h,\ldots,a\sim b_s$, $b_{s+1}\succ a,\ldots,b_{t+1}\succ a$. According to the descending rule, the highest index ϕ such that a is indifferent to b_ϕ is $\phi=s$. Since $\sigma(a,b_{s+1})<\sigma(b_s,a)$, then class C_s is selected for the assignment of a. According to the ascending rule, the lowest index φ such that a is indifferent to b_φ is $\varphi=h$. Since $\sigma(a,b_h)>\sigma(b_{h-1},a)$, C_h is selected for the assignment of a. Consequently, a is assigned to a range of classes $[C_h,C_s]$ (see Table 3 (column VIII)). If h=s, then a is assigned precisely to C_h (see Table 2 (column IV)).
3. If a is *incomparable to at least one characteristic profile*, then $\exists h\in\{0,\ldots,t-1\}$, $\exists s\in\{h,\ldots,t\}$: $a\succ b_0$, $a\succ b_1,\ldots,a\succ b_{h-1}$, $aRb_h,\ldots,aRb_s$, $b_{s+1}\succ a,\ldots,b_{t+1}\succ a$. According to the descending rule, the lowest index φ such that a is incomparable to b_φ is $\varphi=h$. Thus, if $\sigma(a,b_h)\geq\sigma(b_{h-1},a)$, then class C_h is selected for the assignment of a; otherwise, C_{h-1} is selected. According to the ascending rule, the highest index ϕ such that a is incomparable to b_ϕ is $\phi=s$. Thus, if $\sigma(a,b_{s+1})\leq\sigma(b_s,a)$, then class C_s is selected for the assignment of a; otherwise, C_{s+1} is selected. Consequently, an alternative a is assigned to one of the following ranges of classes: $[C_h,C_s]$, $[C_h,C_{s+1}]$, $[C_{h-1},C_s]$ (see Table 3 (columns IX–XI)). For the case $h=s$, see Table 2 (columns V–VII). Note that an interval of classes $[C_{h-1},C_{s+1}]$ cannot be indicated by the joint rules, because it is not possible that $\sigma(a,b_h)<\sigma(b_{h-1},a)$ and $\sigma(a,b_{s+1})>\sigma(b_s,a)$ for $h\leq s$.

Appendix B. Proof of correctness of the inference procedures from Section 4.3

B.1. Proof of correctness of the inference procedure for the worst desired class

The following conditions prevent a^* from being assigned to a class $C_{L^{DM}(a^*)-1}$ or worse:

- The descending rule: $c(a^*,b_{L^{DM}(a^*)-1})\geq\lambda$ and $c(a^*,b_{L^{DM}(a^*)})\geq c(b_{L^{DM}(a^*)-1},a^*)$. Note that if $c(a^*,b_{L^{DM}(a^*)-1})<\lambda$, then the class indicated by the descending rule is at most $C_{L^{DM}(a^*)-1}$. Hence $c(a^*,b_{L^{DM}(a^*)-1})\geq\lambda$ is required. Further, even if this condition is satisfied, a^* can be assigned to $C_{L^{DM}(a^*)-1}$ if $c(a^*,b_{L^{DM}(a^*)})<c(b_{L^{DM}(a^*)-1},a^*)$. Hence $c(a^*,b_{L^{DM}(a^*)})\geq c(b_{L^{DM}(a^*)-1},a^*)$ is required.
- The ascending rule: $c(b_{L^{DM}(a^*)-1},a^*)<\lambda$ and $c(a^*,b_{L^{DM}(a^*)})>c(b_{L^{DM}(a^*)-1},a^*)$. Note that if $c(b_{L^{DM}(a^*)-1},a^*)\geq\lambda$, then the class indicated by the ascending rule is at most $C_{L^{DM}(a^*)-1}$. Hence $c(b_{L^{DM}(a^*)-1},a^*)\geq\lambda$ is required. Further, even if this condition is satisfied, a^* can be assigned to $C_{L^{DM}(a^*)-1}$ if $c(a^*,b_{L^{DM}(a^*)})\leq c(b_{L^{DM}(a^*)-1},a^*)$. Hence $c(a^*,b_{L^{DM}(a^*)})>c(b_{L^{DM}(a^*)-1},a^*)$ is required.

Analysis of Tables 2 and 3 indicates that if a^* is not incomparable to $b_{L^{DM}(a^*)}$, the following conditions are sufficient to prevent a^* from being assigned to a class $C_{L^{DM}(a^*)-1}$ or worse:

$$[c(a^*,b_{L^{DM}(a^*)-1})\geq\lambda] \text{ and } [c(b_{L^{DM}(a^*)-1},a^*)<\lambda] \text{ and } [c(a^*,b_{L^{DM}(a^*)}) > c(b_{L^{DM}(a^*)-1},a^*)]. \tag{B.1}$$

If a^* is incomparable to $b_{L^{DM}(a^*)}$ (i.e., $[\lambda>c(a^*,b_{L^{DM}(a^*)})]$ and $[\lambda>c(b_{L^{DM}(a^*)},a^*)]$), the following conditions need to be satisfied:

$$[c(a^*,b_{L^{DM}(a^*)-1})\geq\lambda] \text{ and } [c(b_{L^{DM}(a^*)-1},a^*)<\lambda] \text{ and } [c(a^*,b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1},a^*)]. \tag{B.2}$$

Combining (B.1) and (B.2) leads to

$$\begin{aligned}&[c(a^*,b_{L^{DM}(a^*)-1})\geq\lambda] \text{ and } [c(b_{L^{DM}(a^*)-1},a^*)<\lambda] \text{ and } \{[c(a^*,b_{L^{DM}(a^*)}) > c(b_{L^{DM}(a^*)-1},a^*)]\\ &\text{or } \{[c(a^*,b_{L^{DM}(a^*)})\geq c(b_{L^{DM}(a^*)-1},a^*)]\\ &\text{and } [\lambda>c(a^*,b_{L^{DM}(a^*)})] \text{ and } [\lambda>c(b_{L^{DM}(a^*)},a^*)]\}\},\end{aligned} \tag{B.3}$$

which is equivalent to (8).

Constraint set $EL(a^*,L^{DM}(a^*))$ corresponds to the set of condition (8). Note that if υ_L=0, then $c(a^*,b_{L^{DM}(a^*)})>c(b_{L^{DM}(a^*)-1},a^*)$ and constraint set $(EL4_{1-3})$ is always satisfied, which is equivalent to its elimination. Otherwise, if υ_L=1, and the following conditions hold:

$$\{[c(a^*,b_{L^{DM}(a^*)})\geq c(b_{L^{DM}(a^*)-1},a^*)] \text{ and } [\lambda>c(a^*,b_{L^{DM}(a^*)})] \text{ and } [\lambda>c(b_{L^{DM}(a^*)},a^*)]\},$$

in which case $(EL3)$ is eliminated.

B.2. Proof of correctness of the inference procedure for the best desired class

The following conditions prevent a^* from being assigned to a class $C_{R^{DM}(a^*)+1}$ or better:

- The descending rule: $c(a^*,b_{R^{DM}(a^*)+1})<\lambda$ and $c(b_{R^{DM}(a^*)},a^*)>c(a^*,b_{R^{DM}(a^*)+1})$. Note that if $c(a^*,b_{R^{DM}(a^*)+1})\geq\lambda$, then the class indicated by the descending rule is at least $C_{R^{DM}(a^*)+1}$. Hence $c(a^*,b_{R^{DM}(a^*)+1})<\lambda$ is required. Further, even if this condition is satisfied, a^* can be assigned to $C_{R^{DM}(a^*)+1}$ if $c(b_{R^{DM}(a^*)},a^*)\leq c(a^*,b_{R^{DM}(a^*)+1})$. Hence $c(b_{R^{DM}(a^*)},a^*)>c(a^*,b_{R^{DM}(a^*)+1})$ is required.
- The ascending rule: $c(b_{R^{DM}(a^*)+1},a^*)\geq\lambda$ and $c(b_{R^{DM}(a^*)},a^*)\geq c(a^*,b_{R^{DM}(a^*)+1})$. Note that if $c(b_{R^{DM}(a^*)+1},a)<\lambda$, then the class indicated by the ascending rule is at least $C_{R^{DM}(a^*)+1}$. Hence $c(b_{R^{DM}(a^*)+1},a^*)\geq\lambda$ is required. Further, even if this condition is satisfied, a^* can be assigned to $C_{R^{DM}(a^*)+1}$ if $c(b_{R^{DM}(a^*)},a^*)<c(a^*,b_{R^{DM}(a^*)+1})$. Hence $c(b_{R^{DM}(a^*)},a^*)\geq c(a^*,b_{R^{DM}(a^*)+1})$ is required.

Analysis of Tables 2 and 3 indicates that if a^* is comparable to $b_{R^{DM}(a^*)}$, the following conditions are sufficient to prevent a^* from being assigned to a class $C_{R^{DM}(a^*)+1}$ or better:

$$[c(b_{R^{DM}(a^*)+1},a^*)\geq\lambda] \text{ and } [c(a^*,b_{R^{DM}(a^*)+1})<\lambda] \text{ and } [c(b_{R^{DM}(a^*)},a^*) > c(a^*,b_{R^{DM}(a^*)+1})]. \tag{B.4}$$

If a^* is incomparable to $b_{R^{DM}(a^*)}$ (i.e., $[\lambda>c(a^*, b_{R^{DM}(a^*)})]$ and $[\lambda>c(b_{R^{DM}(a^*)},a^*)]$), the following conditions need to be satisfied:

$$[c(b_{R^{DM}(a^*)+1},a^*)\geq\lambda] \text{ and } [c(a^*,b_{R^{DM}(a^*)+1})<\lambda] \text{ and } [c(b_{R^{DM}(a^*)},a^*) \geq c(a^*,b_{R^{DM}(a^*)+1})]. \tag{B.5}$$

Combining (B.4) and (B.5) leads to

$$\begin{aligned}&[c(b_{R^{DM}(a^*)+1},a^*)\geq\lambda] \text{ and } [c(a^*,b_{R^{DM}(a^*)+1})<\lambda] \text{ and } \{[c(b_{R^{DM}(a^*)},a^*) > c(a^*,b_{R^{DM}(a^*)+1})]\\ &\text{or } \{[c(b_{R^{DM}(a^*)},a^*)\geq c(a^*,b_{R^{DM}(a^*)+1})]\\ &\text{and } [\lambda>c(a^*,b_{R^{DM}(a^*)})] \text{ and } [\lambda>c(b_{R^{DM}(a^*)},a^*)]\}\},\end{aligned} \tag{B.6}$$

which is equivalent to (9).

Constraint set $EU(a^*,R^{DM}(a^*))$ corresponds to the set of (9). The explanation is analogous to the one of $EL(a^*,L^{DM}(a^*))$ in Appendix B.1.

Appendix C. Inconsistency analysis

If there is no instance of the outranking model compatible with the preference information, i.e. $E(A^R)$ is infeasible or $\varepsilon^* \leq 0$, the DM is asked to revise her preference information. In order to identify reasons of incompatibility, let us associate with each assignment example $a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}]$ a new binary variable v_{a^*}. Using this binary variable, we rewrite the set of constraints $E(a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}])$ as follows:

$$\left.\begin{array}{l}(EL1)' \ M \cdot v_{a^*} + c(a^*, b_{L^{DM}(a^*)-1}) \geq \lambda, \\ (EL2)' \ -M \cdot v_{a^*} + c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon \leq \lambda, \\ \ldots \\ (EU4_3)' \ M \cdot v_{a^*} + M \cdot (1-v_R) + \lambda \geq c(b_{R^{DM}(a^*)}, a) + \varepsilon, \\ (ELU5)' \ v_{a^*}, v_L, v_R \in \{0,1\}.\end{array}\right\} E(a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}])'$$

If $v_{a^*} = 1$, then the corresponding constraint set is satisfied with all parameter values, which is equivalent to its elimination. Identifying a minimal subset of conflicting exemplary assignments can be performed by solving the following MILP problem:

$$\text{Min}: \sum_{a^* \in A^R} v_{a^*}, \qquad \text{(C.1)}$$

s.t.:

$$\left.\begin{array}{l}E(BASE), \\ E(a^* \rightarrow [C_{L^{DM}(a^*)}, C_{R^{DM}(a^*)}])' \text{ for all } a^* \in A^R.\end{array}\right\} E(A^R)'$$

The optimal solution of the above problem indicates the subset of smallest cardinality that is the cause of incompatibility. The other subsets can be obtained following the general scheme for dealing with incompatibility as presented in [44].

Appendix D. Proof of correctness of the inference procedures from Section 4.4

D.1. Proof of correctness of the inference procedure for the possible assignment

If any binary variable v involved in the formulation of $ENL(a,h)$ is 1, then the corresponding condition holds. If $v=0$, the respective constraint is always satisfied, and thus eliminated. $(LP7)$ guarantees that either $(LP1)$ or $(LP2)$ or $(LP3)$ is instantiated with 1. This ensures that one of the following conditions is satisfied:

$c(a, b_h) < \lambda$ or $c(b_h, a) \geq \lambda$ or $c(b_h, a) \geq c(a, b_{h+1})$.

If $c(b_h, a) \geq c(a, b_{h+1})$ was satisfied (i.e., $v_{L3} = 1$), then $(LP8)$ ensures that either $(LP4)$ or $(LP5)$ or $(LP6)$ is instantiated with 1, i.e. one of the following constraints holds:

$c(b_h, a) > c(a, b_{h+1})$ or $c(a, b_{h+1}) \geq \lambda$ or $c(b_{h+1}, a) \geq \lambda$.

Constraint set $ENL(a,h)$ ensures that the set of conditions ensuring that C_{h+1} is the worst class to which an alternative $a \in A$ is assigned to, is not satisfied. Thus, $\neg(L^S(a) \geq h+1) \Rightarrow L^S(a) \leq h$. Analogously, the constraint set $(UP1-9)$ guarantees that C_{h-1} is not the best class which an alternative $a \in A$ is assigned to. Thus, $\neg(R^S(a) \leq h-1) \Rightarrow R^S(a) \geq h$.

The constraint set $E(A^R)$ defines the set of outranking models $\mathcal{S}$ compatible with the DM's preference information. Hence $E(a \rightarrow^P C_h)$ has the necessary constraints for assigning a to the range of classes $[C_{L^S(a)}, C_{R^S(a)}]$, such that $L^S(a) \leq h \leq R^S(a)$, for a compatible outranking model $S \in \mathcal{S}$. If $E(a \rightarrow^P C_h)$ is feasible and $\varepsilon^* = \max \varepsilon$ s.t. $E(a \rightarrow^P C_h) > 0$, then there exists at least one compatible outranking model $S \in \mathcal{S}$ assigning a to C_h. Hence $h \in C_P(a)$.

D.2. Proof of correctness of the inference procedure for the necessary assignment

To check whether a is assigned to a class at least as good as C_{h+1} for some $S \in \mathcal{S}$, we need to consider the set of constraints $E(a \rightarrow^P C_{\geq h+1})$. Constraint set $EL(a, h+1)$ guarantees that a will be assigned to a class $C_{\phi \geq h+1}$. Constraint set $E(A^R)$ defines the set of outranking models $\mathcal{S}$ compatible with the DM's preference information. Hence $E(a \rightarrow^P C_{\geq h+1})$ has the necessary constraints for assigning a to a class better than C_h for a compatible outranking model $S \in \mathcal{S}$. If $E(a \rightarrow^P C_{\geq h+1})$ is feasible and $\varepsilon^* = \max \varepsilon$ s.t. $E(a \rightarrow^P C_{\geq h+1}) > 0$, then $\exists S \in \mathcal{S}$ assigning a to a class worse than C_h. On the contrary, if $E(a \rightarrow^P C_{\geq h+1})$ is infeasible or $\varepsilon^* = \max \varepsilon$ s.t. $E(a \rightarrow^P C_{\geq h+1}) \leq 0$, then all compatible outranking models $S \in \mathcal{S}$ assign a to a class range whose lower bound is not better than C_h.

To verify whether a is assigned to a class worse than C_h for some $S \in \mathcal{S}$, we need to consider set of constraints $E(a \rightarrow^P C_{\leq h-1})$. Constraint set $EU(a, h-1)$ guarantees that a will be assigned to a class $C_{\phi \leq h-1}$. Proceeding analogously to the procedure for verification if $\nexists S \in \mathcal{S}$, such that $L^S(a) \geq h+1$, we conclude that if $E(a \rightarrow^P C_{\leq h-1})$ is infeasible or $\varepsilon^* = \max \varepsilon$ s.t. $E(a \rightarrow^P C_{\leq h-1}) \leq 0$, then all compatible outranking models $S \in \mathcal{S}$ assign a to a class range whose upper bound is not worse than C_h.

If a is neither assigned to a class at least C_{h+1} nor to a class at most C_{h-1} with any compatible outranking model $S \in \mathcal{S}$, then a is assigned to C_h by all $S \in \mathcal{S}$.

Appendix E. Proof of no class jumps-property for necessary assignments of Electre Tri-C

Proof. By the definition of assignment procedures of Electre Tri-C, each compatible outranking model $S \in \mathcal{S}$ provides recommendation formed by the set of contiguous classes $[L^S(a), R^S(a)]$, $\forall a \in A$. Now, if $\bigcap_{S \in \mathcal{S}}[L^S(a), R^S(a)] = \varnothing \Longrightarrow C_N(a) = \varnothing$. Otherwise, if $\bigcap_{S \in \mathcal{S}}[L^S(a), R^S(a)] \neq \varnothing$, the contiguity of $[L^S(a), R^S(a)]$, $S \in \mathcal{S}$, implies that $C_N(a)$ is formed by the set of contiguous classes, i.e. $C_N(a) = [L_N(a), R_N(a)]$. □

Appendix F. Proof of correctness of the reformulated Electre Tri-C assignment rules

F.1. Proof of correctness of the Electre Tri-C assignment rule indicating the worst class

See proof of Theorem 1 on why a is not assigned to a class C_h or worse. The order of verification of the underlying conditions (from the best profile to the worst one) implies that b_h is the best profile for which (15) is satisfied. Thus, C_j, $j > h+1$ is certainly not the worst class a can be assigned to.

F.2. Proof of correctness of the Electre Tri-C assignment rule indicating the best class

See proof of Theorem 2 on why a is not assigned to a class C_h or better. The order of verification of the underlying conditions (from the worst profile to the best one) implies that b_h is the best profile for which (16) is satisfied. Thus, C_j, $j < h-1$ is certainly not the best class a can be assigned to.

Appendix G. Proof of no class jumps-property for possible assignments of Electre Tri-rC

Proof. Proceed by induction on δ^{h}; the base case is $\delta^h = 2$. For $S' \in \mathcal{S}$, a is assigned to class at most C_h iff $b_{h+1} \succ a$ (i.e., $\eta(b_{h+1}, a, S') \geq 0$ and $\eta(a, b_{h+1}, S') < 0$) and $c(b_h, a) > c(a, b_{h+1})$ (see (18)). For $S'' \in \mathcal{S}$, a is assigned to class at least C_{h+2} iff $a \succ b_{h+1}$ (i.e., $\eta(a, b_{h+1}, S'') \geq 0$ and $\eta(b_{h+1}, a, S'') < 0$) and $c(a, b_{h+2}) > c(b_{h+1}, a)$ (see (17)). Now, a is assigned to C_{h+1} if $\neg(a \succ b_{h+1})$ and $\neg(b_{h+1} \succ a)$ or some other conditions are satisfied (see constraint set $E'(a \rightarrow^P C_h)$). $\neg(a \succ b_{h+1})$ and $\neg(b_{h+1} \succ a)$ implies $a \sim b_{h+1}$ or aRb_{h+1}. Since $\mathcal{S}$ is convex, all S^* such that

$$\eta(a, b_{h+1}, S^*) = \alpha\eta(a, b_{h+1}, S') + (1-\alpha)\eta(a, b_{h+1}, S''),$$
$$\eta(b_{h+1}, a, S^*) = \beta\eta(b_{h+1}, a, S') + (1-\beta)\eta(b_{h+1}, a, S''),$$

for $\alpha, \beta \in [0,1]$, are contained in $\mathcal{S}$. Given that $\eta(a, b_{h+1}, S)$ and $\eta(b_{h+1}, a, S)$ are continuous functions that change sign from S' to S'', there must exist $S^* \in \mathcal{S}$, such that at the same time either $\eta(a, b_{h+1}, S^*) \geq 0$ and $\eta(b_{h+1}, a, S^*) \geq 0$ or $\eta(a, b_{h+1}, S^*) < 0$ and $\eta(b_{h+1}, a, S^*) < 0$. In the previous case, $a \sim b_{h+1}$; in the latter case, aRb_{h+1}. These conditions are sufficient for assigning a to class C_{h+1}. This concludes the proof for $\delta^h = 2$. The induction assumption is that the Theorem holds for $\delta^h = \Delta \geq 2$. Now, to prove that the Theorem holds for $\delta^h = \Delta + 1$, proceed analogously to $\delta^h = 2$. □

Appendix H. Inferring additional parameters of the outranking model from assignments examples

In this section, we consider a variant of the outranking model assuming that the indifference q_j and preference p_j thresholds need to be equal, so there is no ambiguity zone. Thus, in what follows, we will refer only to the preference threshold p_j. In order to construct an outranking model, the DM may provide constraints (or, in particular, precise values) concerning characteristic profiles b_h, $h = 1, \ldots, t$, and preference thresholds p_j, $j = 1, \ldots, n$. Additionally, she may define constraints on the weights w_j, $j = 1, \ldots, n$, and on the majority threshold λ.

The binary variables $\psi_j(x, y)$, $j = 1, \ldots, n$, represent the partial concordance indices such that $\psi_j(x, y) = 1$ if the performance of alternative x on g_j is not worse than the performance of the characteristic profile y by more than the preference threshold p_j. The continuous variables $c_j(x, y)$ represent the weighted partial concordance indices such that $c_j(x, y) = w_j$ if $\psi_j(x, y) = 1$, and $c_j(x, y) = 0$, otherwise. Consequently, the base model is defined with the following set of constraints:

$$\left.\begin{array}{l}
\sum_{j=1}^{n} w_j = 1,\\
\sum_{j=1}^{n} c_j(b_h, b_{h+1}) + \varepsilon \leq \lambda, \;\; h = 1, \ldots, t-1,\\
0.5 \leq \lambda \leq 1.0,\\
\text{for } j = 1, \ldots, n,\\
0 \leq w_j \leq 1,\\
g_j(b_{h-1}) \leq g_j(b_h), \quad h = 1, \ldots, t+1,\\
\psi_j(x, y) \geq 1/M \cdot [(g_j(x) - g_j(y) + p_j) + \varepsilon],\\
\psi_j(x, y) \leq 1/M \cdot [g_j(x) - g_j(y) + p_j] + 1,\\
0 \leq p_j \leq g_j(b_{t+1}) - g_j(b_0),\\
c_j(x, y) \leq w_j,\\
c_j(x, y) \geq 0,\\
c_j(x, y) \leq \psi_j(x, y),\\
c_j(x, y) \geq \psi_j(x, y) + w_j - 1,\\
\psi_j(x, y) \in \{0, 1\},
\end{array}\right\} E(BASE)$$

where M is an arbitrary positive large value ensuring that $-1 < 1/M \cdot [g_j(x) - g_j(y) + p_j] < 1$ and ε is an arbitrary small positive value.

Appendix I. Inferring veto thresholds from assignment examples

The approach presented in this paper remains valid for the original definition of credibility in the Electre Tri-B method [12]. However, the use of such a model in the context of indirect preference information is limited by the weak efficiency of non-linear mixed integer programming solvers. Nevertheless, we can reformulate the inference procedures discussed so that the credibility degree is defined as in Section 2. For the sake of brevity, let us present only the equivalent form of $EL(a^*, L^{DM}(a^*))$ (the other constraint sets are reformulated similarly):

$$\left.\begin{array}{l}
(EL1_1^V)\; c(a^*, b_{L^{DM}(a^*)-1}) \geq \lambda\\
(EL1_2^V)\; g_j(b_{L^{DM}(a^*)-1}) - g_j(a^*) + \varepsilon \leq v_j, \quad j = 1, \ldots, n,\\
(EL2_1^V)\; c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon \leq \lambda + M_0(b_{L^{DM}(a^*)-1}, a^*),\\
(EL2_2^V)\; g_j(b_{L^{DM}(a^*)-1}) - g_j(a^*) \geq v_j - \delta M_j(b_{L^{DM}(a^*)-1}, a^*),\\
(EL2_3^V)\; \sum_{j=0}^{n} M_j(b_{L^{DM}(a^*)-1}, a^*) \leq n,\\
(EL2_4^V)\; M_j(b_{L^{DM}(a^*)-1}, a^*) \in \{0, 1\}, \quad j = 0, \ldots, n,\\
(EL3_1^V)\; M \cdot v_L + c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*) + \varepsilon - M_1,\\
(EL3_2^V)\; g_j(b_{L^{DM}(a^*)}) - g_j(a^*) + \varepsilon \leq v_j, \quad j = 1, \ldots, n,\\
(EL3_3^V)\; \sum_{j=1}^{n} M_j(b_{L^{DM}(a^*)-1}, a^*) \leq n - 1 + M_2.\\
(EL3_4^V)\; M_1 + M_2 \leq 1,\\
(EL4_{11}^V)\; M \cdot (1 - v_L) + c(a^*, b_{L^{DM}(a^*)}) \geq c(b_{L^{DM}(a^*)-1}, a^*) - M_3,\\
(EL4_{12}^V)\; g_j(b_{L^{DM}(a^*)}) - g_j(a^*) + \varepsilon \leq v_j + \delta M_3, \;\; j = 1, \ldots, n,\\
(EL4_{13}^V)\; M_2 + M_3 \leq 1,\\
(EL4_{21}^V)\; M \cdot (1 - v_L) + \lambda \geq c(a^*, b_{L^{DM}(a^*)}) + \varepsilon - M_0(a^*, b_{L^{DM}(a^*)}),\\
(EL4_{22}^V)\; g_j(b_{L^{DM}(a^*)}) - g_j(a^*) \geq v_j - \delta M_j(a^*, b_{L^{DM}(a^*)}),\\
(EL4_{23}^V)\; \sum_{j=0}^{n} M_j(a^*, b_{L^{DM}(a^*)}) \leq n,\\
(EL4_{24}^V)\; M_j(a^*, b_{L^{DM}(a^*)}) \in \{0, 1\}, \quad j = 0, \ldots, n,\\
(EL4_{31}^V)\; M \cdot (1 - v_L) + \lambda \geq c(b_{L^{DM}(a^*)}, a^*) + \varepsilon - M_0(b_{L^{DM}(a^*)}, a^*),\\
(EL4_{32}^V)\; g_j(a^*) - g_j(b_{L^{DM}(a^*)}) \geq v_j - \delta M_j(b_{L^{DM}(a^*)}, a^*),\\
(EL4_{33}^V)\; \sum_{j=0}^{n} M_j(b_{L^{DM}(a^*)}, a^*) \leq n,\\
(EL4_{34}^V)\; M_j(b_{L^{DM}(a^*)}, a^*) \in \{0, 1\}, \quad j = 0, \ldots, n,\\
(EL5^V)\; v_L, M_1, M_2, M_3 \in \{0, 1\},
\end{array}\right\} EL(a^*, L^{DM}(a^*))_V$$

where δ is an arbitrary positive large value.

In the original formulation the constraint $(EL1)$ guaranteed that the concordance test for an ordered pair $(a^*, b_{L^{DM}(a^*)-1})$ is positive. After reformulation, $(EL1^V)$ ensures that both concordance and non-discordance tests are positive, which means that concordance index $c(a^*, b_{L^{DM}(a^*)-1})$ is required to be $\geq \lambda$, and that a^* is not significantly worse than $b_{L^{DM}(a^*)-1}$ on any criterion. On the other hand, the original constraint $(EL2)$ guaranteed that the concordance test for an ordered pair $(b_{L^{DM}(a^*)-1}, a^*)$ is negative. Instead, constraint set $(EL2^V)$ ensures that either concordance index $c(b_{L^{DM}(a^*)-1}, a^*)$ is required to be less than a majority threshold λ, or there is at least one criterion g_j for which a^* is evaluated better than $b_{L^{DM}(a^*)-1}$ by more than a veto threshold v_j. Other constraints are reformulated similarly.

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